Transport contract optimization under information asymmetry: an example

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ABSTRACT. Transport cost is second in importance after production cost in industry. It is the purpose of the present paper to solve the single period, single echelon, shipper-carrier transport model established in an earlier paper when demand addressed to the shipper and the spot transport price, two exogenous stochastic variables, follow a bivariate exponential probability distribution function. We evaluate the objective functions of the carrier and shipper over one period reiterated with a mix of long-term and short-term procurement strategies under four scenarios of information sharing. We provide optimal parameters for the shipper's procurement policy where transport capacity needed by the shipper and spot price of transport follow exponential probability distribution functions. We study the behaviour of the different variables versus each other. [1]

Keywords:

Supply chain management; contracts; transport; 3PL; coordination.

1. INTRODUCTION

In this paper, we consider as established by Brusset (2004) that the best transport procurement policy between a shipper and a carrier is one where spot and long term contracts are used simultaneously. The contract involves a base capacity at a set price and includes a menu of prices for some pre-agreed levels of additional capacity offered by the carrier to the shipper should the shipper require more than the base capacity. Also set in the contract are the penalties that each party can ask for compensation from the other: one is when the shipper cannot take all the base capacity that she has committed the carrier to. The other is when the carrier cannot meet the capacity requirements from the shipper. Apart from the contract, the

[1] Special acknowledgment to Per Agrell (agrell@core.ucl.ac.be), Université Catholique de Louvain, Center of Operational Research and Econometrics, Belgium for his inspiring comments.

shipper can decide to purchase the needed capacity from the spot market or from the carrier at the spot market price of the moment.

This paper is organised as follows. In the next section we describe the model involving one single tier in the supply chain: the contractual relationship between one shipper as client and one carrier as transport supplier. In the third section we describe the information asymmetries that both shipper and carrier may face through four scenarios of behaviour for them: in the first, base scenario, the information is common to both, decisions are centrally coordinated. In the second scenario, we consider that the carrier retains a certain measure of information from the shipper. In the third, both shipper and carrier hide information from each other. In the fourth, the shipper has private information on her received demand. We enunciate the necessary objective functions. In the fourth section, we solve for the optimal contract characteristics according to an example of a bivariate exponential distribution function involving demand addressed to the shipper and spot market price for capacity. Finally, we draw conclusions from the results.

2. Transport Market Model

3.1.1. *Basic contract*

C (carrier) and S_L (shipper) have negotiated ex-ante and are bound by a contract with known and fixed parameters. S_L agrees to buy base capacity q_L at price c. The shipper has to pay a penalty θ_s for unused capacity up to q_L at each period. The carrier suffers a penalty θ_c if he cannot (or chooses no to) carry the contracted capacity q_L at each period for non-performance of contracted service.

3.1.2. *Menu of prices for additional capacity*

The contract includes a menu of prices ranging up to p_{La} for quantities up to a maximum of q_{La} that the carrier offers to the shipper S_L to help him meet demand in excess of the contracted capacity commitment q_L (fig.1). The menu is a list of prices in linear function with the capacity offered. This seems counter intuitive: one would expect that the higher the capacity sought by the shipper, the less the marginal cost to the carrier, so that the carrier would be motivated to make a volume discount to capture the excess demand. We have established earlier that this follows earlier proofs of better efficiency to the supply chain.

3.2. *Opportunistic behaviour*

Opportunistic behaviour occurs when either the shipper S_L or the carrier can escape from their contractual engagements without incurring retaliation from the other party. All retaliation depends upon verifying opportunistic behaviour which bears a cost. We will focus in this paper on certain pieces of information which can make a significant impact on the cost functions of either party. We centre our attention here on two particularly sensitive pieces of information.

3.2.1. *Transport capacity of carrier*

The first piece of information is the size of the transport capacity the carrier owns or otherwise controls. Ex-ante the shipper verifies the available capacity of the carrier and the carrier must convey all necessary information so that the shipper can be assured that the required capacity exists. Thereafter, no further control is undertaken by the shipper. So, in the course of the life of the contract, this information is no longer observable. Only when the contract comes up for renewal can the shipper use records of past shipments to assess the capacity of the carrier.

3.2.2. *Available cargo to be shipped*

The second piece of information involves the size of the available transport requirements of the shipper: the carrier cannot verify period after period that the orders handed him by the shipper represent his entire need. This information is also neither observable directly nor verifiable without cost to the carrier. The shipper may contract added capacity with other carriers whenever its suits him financially.

3.2.3. *Covert actions by each actor*

The covert actions that can be engaged in are of four types.

Depending upon the spot price in the market and since the carrier's total capacity is non-verifiable and non-observable, the carrier can engage in hidden action (shirk his contractual engagements) by refusing to comply with the demand from the shipper, pay the corresponding penalty θ_c and sell this excess capacity in the spot market. In this case, the shipper has to buy transport capacity from the spot market.

The shipper S_L can also engage in hidden action when her realised demand is not observable by the carrier. When the spot price is lower than the menu of prices less the penalty, the shipper is better off by refusing to purchase capacity in excess of q_L from the carrier and buy instead the necessary complement from the spot market, since the demand is unknown, the carrier cannot demand nor obtain compensation for lost business from S_L.

The fourth possible scenario is when the shipper knows the capacity of the carrier and the carrier is not aware of the exact demand received by the shipper. The carrier cannot shirk his contractual engagements but the shipper can contract additional capacity from the spot market when the spot price is lower than the menu of prices for additional capacity. She would not have to pay a penalty to the carrier since the carrier is unaware of the extra cargo to ship.

3. LITERATURE REVIEW

Supply chain performance depends critically on how its members coordinate their decisions and it is hard to imagine coordination without some form of information sharing, as Fangruo Chen remarked in Chen (2002). In the supply chain management literature, transport service providers as suppliers are not usually individualised as such. One line of literature research focuses on efficient planning of routes, networks, warehouse location etc. In the other line of literature, supply chain efficiency can be increased by coordination, truth-inducing mechanisms, contractual engineering and information sharing (see Chen's review of 2002 and 1998, 2001a, 2001b, Chen & Yu 2001a, Chen – Yu 2001b, Anupindi 1998, Porteus and Whang 1991, Lee and Whang 1999, Cachon et al. 1999, Zhao 2002). However, since their supplier definition entails back-logging of orders and inventory management, not all results apply to carriers or shippers. Ertogral et al. (1998) bridges both lines of thought: a single model integrates production and transportation planning, taking into account transport costs and schedules. This approach does not take into account the impact of imperfect information and decentralised decisions. Neither does it take into consideration the eventual over or under utilisation of the transport capacity involved.

Full capacity utilisation is one of the primary objectives of the transport industry. Gilbert and Cvsa (2000) model the behaviour of the client who under-invests in cost-reduction so as to escape being held up by the supplier. By crafting an ex-ante pricing contract, the retailer's interests are aligned with the manufacturer's and this eliminates or at least reduces supplier opportunistic behaviour.

Wu et al (2002) has modelled contracting arrangements for capital-intensive, capacity-constrained goods in the energy sector. They highlight the "two-goods problem": capacity itself, pre-committed to a specific buyer, and output actually delivered on the day to the buyer. This gives rise to two different markets where prices are formed: spot pricing and pre-arranged bilateral contracting. The paper provides valuable insights on the optimal balance between selling capacity in the forward contract market versus selling on the spot market. This is not exactly the case of transport: parties to a contract have to iron out several operational details regarding execution, quality criteria, etc that make each contract unique and entails greater transaction costs.

Spinler & Huchzermeier (2003) propose a variation of the preceding model by using options in lieu of future and spot market contracts to increase capacity utilisation in the presence of state-contingent demand. They show that such a strategy effectively is Pareto improving things for both the seller of the option (transport supplier) and the buyer (the shipper). To circumvent the liquidity problem of transport as a non-standardised service, the model assumes that options will be traded on electronic marketplaces. However, as Grieger (2003) reported (Cf. supra), carriers and shippers may be wary to trade with partners of unknown quality and customer service levels.

Agrell et al (2002) model a 3-stage, 2-period supply chain in the telecom sector. A supplier can decide to invest in certain new capacity and may share the economies by lowering his price. This model specifically excludes long-term partnerships that encourage parties to engage in activities that are unfavourable in the short term but have substantial payoffs over time.

We draw on the quantity flexibility contract clause mechanism under retailer uncertain demand (Tsay et al. 1999a, Tsay 1999, Tsay & Lovejoy 1999, Anupindi 1998, Tsay 2000, Cachon 2002), designed to align the behaviour of the supplier. "The Quantity Flexibility clause defines terms under which the quantity a buyer ultimately obtains may deviate from a previous planning estimate. The conditions can include limits on the range of allowable changes, pricing rules, or both." (Tsay et al 1999a).

The shipper must reduce capacity cost for a given demand risk. In other words, he must offload the risk onto the carrier. Some measure of flexibility in capacity has to be introduced. One mechanism would be to set up a menu of extra capacities at pre-arranged prices: if the demand effectively exceeds the base contractual capacity, the shipper calls up extra capacity to meet it using this clause to set the premium price. Another would be to set a penalty clause for the carrier when he is unable to meet the capacity thus committed: whenever the carrier fails to meet the shipper's demand, he pays a penalty proportionate to the shortcoming. In Moinzadeh & Nahmias 1997 that same general problem is treated: Q, the minimum commitment per period is given and there are both fixed and proportional penalties for adjustments, over an infinite horizon. The authors contend, but do not formally prove, that a type of order-up-to policy (s,S) is optimal. In that model, the fixed delivery contract with penalties serves as a risk sharing mechanism.

Because the demand, when realised, directly results in a transport requirement, there can be no time-flexibility arrangements as those described in the literature (Li & Kouvelis 1997, Barnes & Schuster 1997).

In our approach, transport capacities are not freely substitutable, ruling out "overbooking" (Karaesmen et al., 2002). Moreover, there are few cases of "no-shows".

Our market mechanism draws also on the model in Seifert et al. (2003) for simultaneous long-term and short-term (spot) buying of commodities by a client from one or various suppliers. The shipper can simultaneously buy through long term contracts and through spot transactions the needed transport capacity.

Our model follows a similar pattern to that adopted in Gavirneni et al. (1999): three scenarios that differ by the information level of the participants are studied.

4. TRANSPORT MARKET MODEL

3.2.4. *Basic contract*

C and S_L have negotiated ex-ante and are bound by a contract extending over one period of n sub periods with known and fixed parameters. S_L agrees to buy at each sub-period capacity q_L at price c. The shipper has to pay a penalty θ_s for unused capacity up to q_L at each period. The carrier suffers a penalty θ_c if he cannot (or chooses no to) carry the contracted capacity q_L at each period for non-performance of contracted service.

3.2.5. *Menu of prices for additional capacity*

The contract includes a menu of prices p_{La} at quantities q_{La} that the carrier offers to the shipper S_L to help him meet demand in excess of the contracted capacity commitment q_L up till q_{La} (fig.1). The menu is a list of prices linear with the capacity offered. This seems counter intuitive: one would expect that the higher the capacity sought by the shipper, the less the marginal cost to the carrier, so that the carrier would be motivated to make a volume discount to capture the excess demand. We will revisit this matter when discussing the coordinating power of the contract. Each price in the menu is the going price for all the excess capacity required by the shipper. This menu is not a menu of options in the true sense since there is no premium to be paid but rather an option on a forward contract as the shipper is committed to taking the available capacity offered under the terms of the menu (quantity and price); even if the spot price is less than the price in the menu for that given additional capacity.

3.3. *Opportunistic behaviour*

Opportunistic behaviour occurs when either the shipper S_L or the carrier can escape from their contractual engagements without incurring retaliation from the other party. All retaliation depends upon verifying opportunistic behaviour. The party wishing that verification incurs a cost to perform this information collection process. We will focus in this paper on certain pieces of information which can make a significant impact on the cost functions of either party.

The first piece of information is the size of the transport capacity the carrier owns or otherwise controls. Ex-ante the shipper verifies the available capacity of the carrier and the carrier must convey all necessary information so that the shipper can be assured that the required capacity exists. Thereafter, no further control is undertaken by the shipper. So, in the course of the life of the contract, this information is no longer observable. Only when the contract comes up for renewal can the shipper use records of past shipments to assess the capacity of the carrier.

The second piece of information involves the size of the available transport requirements of the shipper: the carrier cannot verify period after period that the orders handed him by the shipper represent his entire need. This information is also neither observable directly nor verifiable without cost to the carrier. The shipper may contract added capacity with other carriers whenever its suits him financially.

Depending upon the spot price in the market and since the carrier's total capacity is non-verifiable and non-observable, the carrier can engage in hidden action by refusing to comply with the demand from the shipper, pay the corresponding penalty θ_c and sell this excess capacity in the spot market. In this case, the shipper has to buy transport capacity from the spot market.

The shipper S_L can also engage in hiden action when her realised demand is not observable by the carrier. When the spot price is lower than the menu of prices less the penalty, the shipper is better off by refusing to purchase capacity in excess of q_L from the carrier and instead buy the necessary complement from the spot market.

We have modelled all four deviations in our scenarios.

3.4. *Demand and capacity characteristics*

State of nature is represented using three variables: P_s is the market price for immediate transport. The demands that the shipper and the market meet individually and non-competitively are two exogenous, stochastic variables ζ_L ζ_s. $\Omega(P_s,Q_L,Q_s)$ is the probability space containing the possible realisations of the triplets of transport spot price, and of demands addressed to shipper S_L and the market. $F_L(.)$ and $F_s(.)$ are the continuously differentiable, invertible and monotonous cumulative distribution functions of demand addressed to, respectively, S_L and the market. $f_L(.)$ (mean μ_L, standard deviation σ_L)and $f_s(.)$ (mean μ_s and standard deviation σ_s) are the density functions of $F_L(.)$ and $F_s(.)$. $F_p(.)$ is the continuously differentiable, invertible and monotonous cumulative distribution function of the spot market price and $f_p(.)$ its density function (mean μ_p and standard deviation σ_p). Let **Error! Objects cannot be created from editing field codes.** be the correlation factor between $F_L(.)$ and $F_s(.)$ and let $\rho_p \in [0,1]$ be the correlation factor between $F_L(.)$ and $F_p(.)$. The shipper S_L knows ex-ante the mean μ_L and standard deviation σ_L of the cumulative distribution function of demand. There is no backlogging of demand: any unsatisfied demand is lost.The demands have to be satisfied in full at each period.

All other production costs of S_L are ignored.

The total capacity of C is W. C has a variable cost per unit transported V_c and a fixed cost F_c. No assumption is made regarding W. F_c is a function of this capacity W.

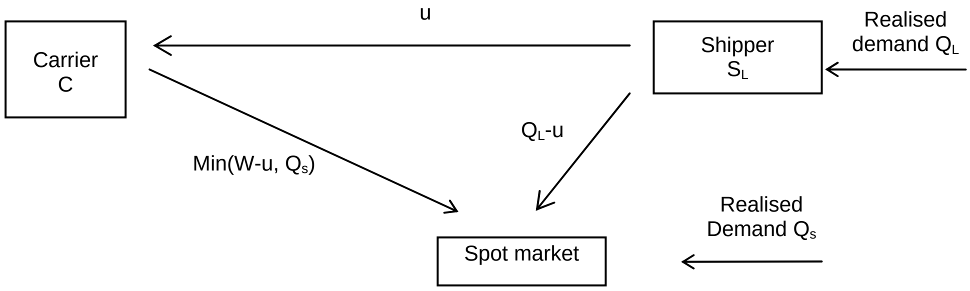

Fig. 1: Capacity allocation

In figure 1, u is the demand that S_L chooses to allot to C.

3.5. *Objective functions*

3.5.1. *Regionalizing the probability space*

We divide the probability space Ω into regions so as to facilitate the discussion regarding the best decision by both S_L and C (fig. 3):

$$
\begin{aligned}
&\Omega1(Q_L,P_s)=\{Q_L:0\le Q_L\le q_L;0\le P_s\}\\
&\Omega2(Q_L,P_s)=\{Q_L:q_L<Q_L;P_s:0\le P_s\le p_{La}-\theta_s\}\\
&\Omega3(Q_L,P_s)=\{Q_L:q_L<Q_L\le q_L+q_{La};P_s:p_{La}-\theta_s<P_s\le p_{La}+\theta_c\}\\
&\Omega4(Q_L,P_s)=\{Q_L:q_L<Q_L\le q_L+q_{La};P_s:p_{La}+\theta_c<P_s\}\\
&\Omega5(Q_L,P_s)=\{Q_L:q_L+q_{La}<Q_L\le W;0\le P_s\le p_{La}-\theta_s\}\\
&\Omega6(Q_L,P_s)=\{Q_L:q_L+q_{La}<Q_L\le W;p_{La}-\theta_s<P_s\le p_{La}+\theta_c\}\\
&\Omega7(Q_L,P_s)=\{Q_L:q_L+q_{La}<Q_L\le W;p_{La}+\theta_c<P_s\}\\
&\Omega8(Q_L,P_s)=\{Q_L:W<Q_L;0\le P_s\}\\
&\Omega9(Q_L,P_s)=\{Q_L:W<Q_L;p_{La}-\theta_s<P_s\le p_{La}+\theta_c\}\\
&\Omega10(Q_L,P_s)=\{Q_L:W<Q_L;p_{La}+\theta_c<P_s\}
\end{aligned}
\tag{1.1}
$$

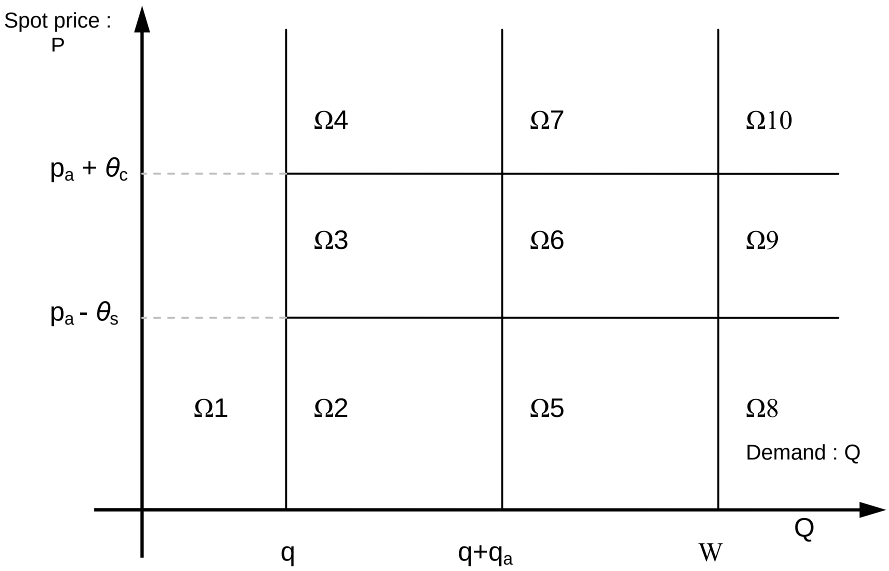

Fig. 3: Probability spaces for spot price and demand addressed to SL

3.5.2. *Carrier objective function*

In our setting, carrier C has just two customers: S_L and the spot market (fig. 1). If the summed demands from these two do not reach total capacity, the excess capacity is lost for all intents and purposes, impacting the carrier's profitability

The objective function of the carrier is to increase revenue and maximise profit.profits. His decision variables are the capacity he allots to each source of demand: x_L is the allotted capacity to S_L and x_s to the spot market. $W-x_L-x_s$ is the wasted capacity. We consider that the fixed costs of supporting the necessary assets are specific, sunk and that the carrier does not have the choice to withdraw from the allocation game with S_L. We therefore neglect all considerations as to fixed costs of C. His profit function can thus be written by using the terms of the contract.

We restate here all the contract characteristics as defined above:

$W \geq x_L + x_s$	Total transport capacity of C (fleet capacity)
$0 < q_L + q_{La} \leq W$	contracted capacity plus negotiated additional capacity has to be less than total capacity
$0 \leq \theta_s < c,$ $0 \leq \theta_c < c$	penalties paid by shipper or carrier are less than the contract price
$0 \leq q_{La} < q_L,$	additional capacity is not higher than the base capacity contracted
$c \leq p_{La}$	price for additional capacity is higher than the base capacity price
$0 \leq u, \leq Q_L$	u is a capacity, decision variable of the shipper is at most equal to total demand received by shipper.
$0 \leq x_L \leq u$	x_L, decision variable of the carrier is at most equal to the effective capacity that the shipper asks him to provide.
$0 \leq x_s \leq Q_s,$	the decision variable of the carrier as to the demand received from the spot market is bounded by the demand received from the spot market and the remaining capacity at his disposal after serving S_L.
$0 \leq P_s,$	the spot price for transport capacity cannot take negative values.

The profit function is conditional upon the allocation by S_L and the spot market price:

$$\pi(x_L \mid u, \Omega_i) = R_i(x_L \mid \Omega i) + P_s x_s - VC(x_L + x_s) \tag{1.2}$$

Where $VC(x_L, x_s)$ is the variable cost as a function of the allocation of capacity to S_L and the market and where R_i is a revenue function, conditional upon the demand u addressed by S_L and the spot market price, of the form:

$$R_i(x_L \mid u, \Omega i) = \begin{cases} x_L c - (\min(u, q_L) - x_L)\theta_c + (q_L - u)\theta_s & : 0 \leq x_L < q_L \\ q_L c + (x_L - q_L) p_{La} & : q_L \leq x_L \leq q_L + q_{La} \\ q_L c + (x_L - q_L) p_{La} + (x_L - q_L - q_{La}) P_s & : q_L + q_{La} < x_L \leq W \end{cases} \tag{1.3}$$

q_L, q_{La}, c, p_{La} and θ_c ,θ_s are the parameters defined by the contract. P_s is the spot market price (fig.4).

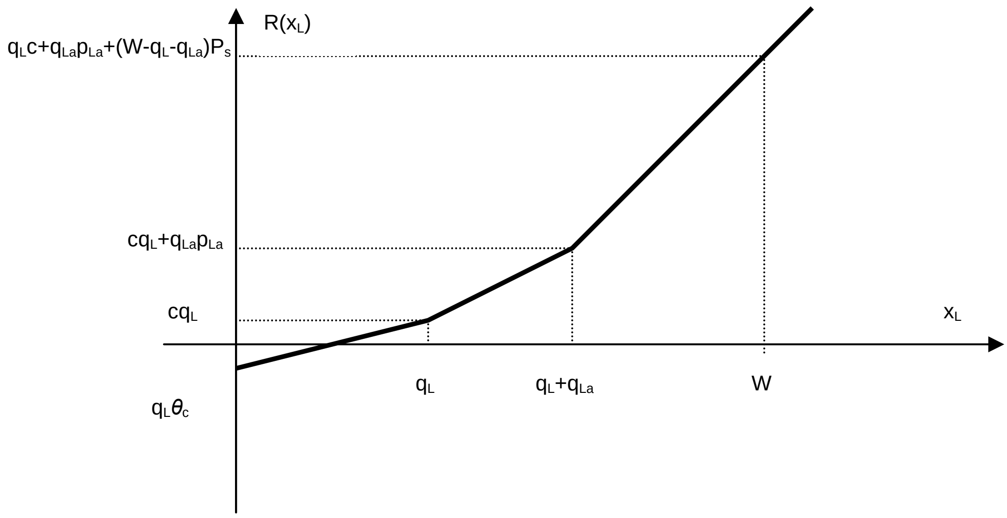

Fig 4: Behaviour of f(xL)where u=W

3.5.3. *Shipper objective function*

Shipper S_L produces and sells a product that requires transportation, either in the internal supply chain process or in the relationship with her clients. She, as Stackelberg leader, must decide whether to allocate her necessity to her chosen contractual carrier at the ex-ante contractual price or to the spot market at the going spot market price.

The decision variable u can take all values between 0 and total received demand Q_L (see fig. 5). Whatever transport necessity is not being allocated to C will be offered to the spot market at the going spot price P_s. The function is conditional upon the response S_L receives from C, which is represented by $x_L(u)$. By investigation, we see also that S_L has an opportunity to reduce transport cost by diverting cargo to the spot market when conditions of the spot price relative to the contract parameters warrant it.

$$C_i\left(u \mid x_L,\Omega i\right)=\begin{cases} c x_L(u)+\left[q_L-u\right]^{+}\theta_s+\left(\min\left(q_L,u\right)-x_L(u)\right)\theta_c+\left(Q_L-x_L(u)\right)P_s & :0\le x_L(u)\le q_L \\ cq_L+\left(x_L(u)-q_L\right)p_{La}-\left[u-x_L(u)\right]^{+}\theta_c+\left[Q_L-u\right]^{+}\theta_s+\left(Q_L-x_L(u)\right)P_s & :q_L<x(u)\le q_L+q_{La} \\ cq_L+\min\left(\left(x_L(u)-q_L\right),q_{La}\right)p_{La}-\min\left(\left[u-x_L(u)\right]^{+},q_{La}\right)\theta_c+\min\left(\left[Q_L-u\right]^{+},q_{La}\right)\theta_s & \\ +\left(Q_L-x_L(u)\right)P_s & :q_L+q_{La}<x(u) \end{cases} \quad (1.4)$$

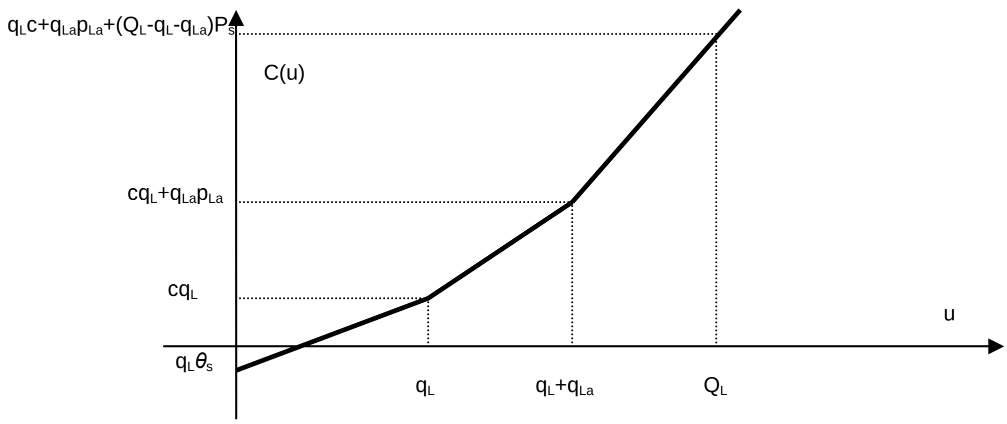

Fig.5: Behaviour of C(u)

3.5.4. *Defining optimal decisions according to demand and spot price*

In each region of probability space, the optimal decisions by each player vary. Let us call R_Ω and C_Ω the revenue and cost functions over each separate region identified by its number.

Case 1: $\Omega 1, Q_L \le q_L, P_s \in]0,\infty[$

Realized demand received by S_L is less than contracted, whatever the spot price, the shipper and carrier honour the contract. Because the minimum capacity included in the contract is higher, S_L pays a penalty in proportion to the unused capacity. Neither have any liberty of choice in this space. C has no choice but to allocate $x_L = u$ capacity to S_L. The equation for this small segment of realized demand and spot price can be written as:

$$R_{\Omega 1}(x_L \mid u,\Omega 1) = x_L(u)c + (q_L - x_L(u))\theta_s$$
$$R_{\Omega 1}(x_{L1}^* \mid u,\Omega 1) = uc + (q_L - x_L(u))\theta_s \quad (1.5)$$

By symmetry, we have:

$$C_{\Omega 1}(u \mid x_L,\Omega 1) = uc + (q_L - x_L(u))\theta_s \quad (1.6)$$

This gives the optimal allocation for S_L: $u_1^* = Q_L$

$$C_{\Omega 1}(u_1^* \mid x_L,\Omega 1) = Q_L c + (q_L - Q_L)\theta_s, \quad (1.7)$$

which minimizes the penalty to be paid to C. Giving in turn: $x_{L1}^* = Q_L$

$$R_{\Omega 1}(x_{L1}^*, u_1^* \mid \Omega 1) = Q_L c + (q_L - Q_L)\theta_s \quad (1.8)$$

Case 2: $\Omega 2, q_L < Q_L \le q_L + q_{La}, 0 < P_s \le p_{La} - \theta_s$

Realized demand addressed to S_L is between q_L and q_{La}. Here, each has the possibility to choose how to allocate capacity above the minimum contractual obligations. S_L can choose to allocate all demand in excess of q_L either to the spot market, to C but at spot price or to C using the menu of prices included in the contract. C also can choose to refuse the extra capacity. The optimal behaviour here is for S_L to call on C for extra capacity but at the spot price, since the spot price is lower than the menu price p_{La} less the penalty for not using carrier C's capacity, and pays to C the penalty θ_s as built into the contract. This reduces S_L's transport cost.

The equation for this segment of realized demand and price is:

$$R_{\Omega 2}(x_L \mid u,\Omega 2) = q_L c + (x_L(u) - q_L)p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s$$
$$C_{\Omega 2}(u \mid x_L,\Omega 2) = q_L c + (x_L(u) - q_L)p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s \quad (1.9)$$

The optimal allocation for each is achieved when $u_2^* = q_L = x_{L2}^*$ and gives:

$$R_{\Omega 2}(x_{L2}^*, u_2^* \mid \Omega 2) = q_L c + (Q_L - q_L)(P_s + \theta_s)$$
$$C_{\Omega 2}(u_2^*, x_{L2}^* \mid \Omega 2) = q_L c + (Q_L - q_L)(P_s + \theta_s) \quad (1.10)$$

Case 3: $\Omega 3, q_L < Q_L \le q_L + q_{La}, p_{La} - \theta_s < P_s \le p_{La} + \theta_c$

Here demand addressed to S_L is still between q_L and q_{La} but the spot price is between p_{La} - θ_s and p_{La} + θ_c. The optimal allocation, is for S_L to use the additional capacity q_{La} at the additional price p_{La}. The optimal behaviour for is to C provide the necessary capacity. Transport is performed. No penalties are due.

The equation is thus:

$$\begin{aligned} R_{\Omega 3}(x_L \mid u, \Omega 3) &= q_L c + (x_L(u) - q_L) p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s \\ C_{\Omega 3}(u \mid x_L, \Omega 3) &= q_L c + (x_L(u) - q_L) p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s \end{aligned} \quad (1.11)$$

The optimal is $u_3^* = Q_L = x_{L3}^*$ and the functions become:

$$\begin{aligned} R_{\Omega 3}\left(x_{L3}^*, u_3^* \mid \Omega 3\right) &= q_L c + (Q_L - q_L) p_{La} \\ C_{\Omega 3}\left(u_3^*, x_{L3}^* \mid \Omega 3\right) &= q_L c + (Q_L - q_L) p_{La} \end{aligned} \quad (1.12)$$

<u>Case 4:</u> $\Omega 4, q_L < Q_L \leq q_L + q_{La}, p_{La} + \theta_c < P_s$

Realized demand comes in between q_L and q_{La} but the spot price is over p_{La} + θ_c. In this case, the carrier will pay S_L a penalty for refusing to carry the additional cargo as per the QF clause. Cargo will still be carried by C but at the spot price: P_s, giving an additional profit to C.

The revenue equation can still be written as in case 3:

$$\begin{aligned} R_{\Omega 4}(x_L \mid u, \Omega 4) &= q_L c + (x_L(u) - q_L) p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s \\ C_{\Omega 4}(u \mid x_L, \Omega 4) &= q_L c + (x_L(u) - q_L) p_{La} - (u - x_L(u))\theta_c + (Q_L - u)\theta_s + (Q_L - x_L(u))P_s \end{aligned} \quad (1.13)$$

But the optimal choice becomes $u_4^* = Q_L, x_{L4}^* = q_L$, leading to:

$$\begin{aligned} R_{\Omega 4}\left(x_{L4}^*, u_4^* \mid \Omega 4\right) &= q_L c + (Q_L - q_L)(P_s - \theta_c) \\ C_{\Omega 4}\left(u_4^*, x_{L4}^* \mid \Omega 4\right) &= q_L c + (Q_L - q_L)(P_s - \theta_c) \end{aligned} \quad (1.14)$$

<u>Case 5:</u> $\Omega 5, q_L + q_{La} < Q_L \leq W, 0 < P_s \leq p_{La} - \theta_s$

In this case, the realized demand exceeds q_L + q_{La} but the spot is lower than the menu of prices less the shipper penalty. The excess over q_L + q_{La} is transported at the spot price, whatever its level. The shipper can lower her cost by allocating capacity between q_L and q_L+q_{La}, The shipper will allocate the minimum possible to C, have the difference transported at P_s and pay the necessary penalty to C. For comparison purposes (within scenarios in the next stage), we have supposed that after refusing to allocate cargo within the contract, S_L still gives the difference of Q_L – q_L to C at the spot price. So the revenue still accrues to C. We have:

$$\begin{aligned} R_{\Omega 5}(x_L \mid u, \Omega 5) &= q_L c + \min\left((x_L(u) - q_L), q_{La}\right) p_{La} - \min\left((u - x_L(u)), q_{La}\right)\theta_c \\ &\quad + \min\left((Q_L - u), q_{La}\right)\theta_s + \min\left((Q_L, W) - x_L(u)\right)P_s \\ C_{\Omega 5}(u \mid x_L, \Omega 5) &= q_L c + \min\left((x_L(u) - q_L), q_{La}\right) p_{La} - \min\left((u - x_L(u)), q_{La}\right)\theta_c \\ &\quad + \min\left((Q_L - u), q_{La}\right)\theta_s + \min\left(Q_L - x_L(u)\right)P_s \end{aligned} \quad (1.15)$$

So that the optimal allocation is the same as in Case 2 $u_5^* = q_L = x_{L5}^*$ and we have:

$$R_{\Omega 5}\left(x_{L5}^{*},u_5^{*}\mid\Omega 5\right)=q_Lc+q_{La}\theta_s+\left(Q_L-q_L\right)P_s$$
$$C_{\Omega 5}\left(u_5^{*},x_{L5}^{*}\mid\Omega 5\right)=q_Lc+q_{La}\theta_s+\left(Q_L-q_L\right)P_s \quad (1.16)$$

Case 6: $\Omega 6, q_L+q_{La}<Q_L\le W, p_{La}-\theta_s<P_s\le p_{La}+\theta_c$

We have the same functions as in case 3 and the optimal is: $u_6^{*}=Q_L=x_{L6}^{*}$

$$R_{\Omega 6}\left(x_{L6}^{*},u_6^{*}\mid\Omega 6\right)=q_Lc+q_{La}p_{La}+\left(Q_L-q_L-q_{La}\right)P_s$$
$$C_{\Omega 6}\left(u_6^{*},x_{L6}^{*}\mid\Omega 6\right)=q_Lc+q_{La}p_{La}+\left(Q_L-q_L-q_{La}\right)P_s \quad (1.17)$$

Case 7: $\Omega 7, q_L+q_{La}<Q_L\le W, p_{La}+\theta_c<P_s$

$$\begin{aligned}R_{\Omega 7}\left(x_L\mid u,\Omega 7\right)=q_Lc&+\min\left(\left(x_L(u)-q_L\right),q_{La}\right)p_{La}-\min\left(\left(u-x_L(u)\right),q_{La}\right)\theta_c\\&+\min\left(\left(Q_L-u\right),q_{La}\right)\theta_s+\left(x_L(u)-q_L\right)P_s\end{aligned} \quad (1.18)$$

$$\begin{aligned}C_{\Omega 7}\left(u\mid x_L,\Omega 7\right)=q_Lc&+\min\left(\left(x_L(u)-q_L\right),q_{La}\right)p_{La}-\min\left(\left(u-x_L(u)\right),q_{La}\right)\theta_c\\&+\min\left(\left(Q_L-u\right),q_{La}\right)\theta_s+\left(x_L(u)-q_L\right)P_s+\left(Q_L-x_L(u)\right)P_s\end{aligned} \quad (1.19)$$

The optimal becomes: $u_7^{*}=Q_L, x_{L7}^{*}=q_L$

The function of this optimal allocation is:

$$R_{\Omega 7}\left(x_{L7}^{*},u_7^{*}\mid\Omega 7\right)=q_Lc+q_{La}\theta_c+\left(Q_L-q_L\right)P_s \quad (1.20)$$

$$C_{\Omega 7}\left(x_{L7}^{*},u_7^{*}\mid\Omega 7\right)=q_Lc+q_{La}\theta_c+\left(Q_L-q_L\right)P_s \quad (1.21)$$

Case 8: $\Omega 8, W<Q_L, 0<P_s\le p_{La}-\theta_s$

In the following three regions of probability space, the overall capacity of C becomes a constraint which limits the revenue to C. C cannot satisfy all S_L's transport capacity. There is no such limit on the cost to S_L since we have supposed that the spot market has unlimited capacity to respond to S_L's needs.

$$R_{\Omega 8}\left(x_{L8}^{*},u_8^{*}\mid 8\right)=q_Lc+q_{La}\theta_s+\left(W-q_L\right)P_s$$
$$C_{\Omega 8}\left(u_8^{*},x_{L8}^{*}\mid\Omega 8\right)=q_Lc+q_{La}\theta_s+\left(Q_L-q_L\right)P_s \quad (1.22)$$

Case 9: $\Omega 9, W<Q_L, p_{La}-\theta_s<P_s\le p_{La}+\theta_c$

$$R_{\Omega 9}\left(x_{L9}^{*},u_9^{*}\mid\Omega 9\right)=q_Lc+q_{La}p_{La}+\left(W-q_L-q_{La}\right)P_s$$
$$C_{\Omega 9}\left(u_9^{*},x_{L9}^{*}\mid\Omega 9\right)=q_Lc+q_{La}p_{La}+\left(Q_L-q_L-q_{La}\right)P_s \quad (1.23)$$

Case 10: $\Omega 10, W<Q_L, p_{La}+\theta_c<P_s$

$$R_{\Omega 10}\left(x_{L10}^{*},u_{10}^{*}\mid\Omega 10\right)=q_Lc+q_{La}\theta_c+\left(W-q_L\right)P_s \quad (1.24)$$

$$C_{\Omega 10}\left(x_{L10}^{*},u_{10}^{*}\mid\Omega 10\right)=q_Lc+q_{La}\theta_c+\left(Q_L-q_L\right)P_s \quad (1.25)$$

The following table recapitulates the results synthetically:

Ωi	u_i^*	x_{Li}^*	$R_{\Omega i}$	$C_{\Omega i}$
Ω1	Q_L	Q_L	$Q_L c+(q_L-Q_L)\theta_s$	$Q_L c+(q_L-Q_L)\theta_s$
Ω2	q_L	q_L	$q_L c+(Q_L-q_L)(P_s+\theta_s)$	$q_L c+(Q_L-q_L)(P_s+\theta_s)$
Ω3	Q_L	Q_L	$q_L c+(Q_L-q_L)p_{La}$	$q_L c+(Q_L-q_L)p_{La}$
Ω4	Q_L	q_L	$q_L c+(Q_L-q_L)(P_s-\theta_c)$	$q_L c+(Q_L-q_L)(P_s-\theta_c)$
Ω5	q_L	q_L	$q_L c+q_{La}\theta_s+(Q_L-q_L-q_{La})P_s$	$q_L c+q_{La}\theta_s+(Q_L-q_L)P_s$
Ω6	Q_L	Q_L	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La})P_s$	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La})P_s$
Ω7	Q_L	q_L	$q_L c+q_{La}\theta_c+(Q_L-q_L)P_s$	$q_L c+q_{La}\theta_c+(Q_L-q_L)P_s$
Ω8	q_L	q_L	$q_L c+q_{La}\theta_s+(W-q_L-q_{La})P_s$	$q_L c+q_{La}\theta_s+(Q_L-q_L)P_s$
Ω9	Q_L	Q_L	$q_L c+q_{La}p_{La}+(W-q_L-q_{La})P_s$	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La})P_s$
Ω10	Q_L	q_L	$q_L c+q_{La}\theta_c+(W-q_L)P_s$	$q_L c+q_{La}\theta_c+(Q_L-q_L)P_s$

Table 1: Expressions of optimal allocation of demand and capacity among different regions of probability space

3.5.5. *Redefining optimal decisions by C to include allocations of capacity to market*

So far, we have dealt with the optimal allocation of demand coming from the contractual shipper S_L. C also has to contend with demand coming from the market at spot prices. This second demand upon his capacity comes second by priority and in time. The discussion of the optimal allocation is guided by the level of the spot price P_s versus the variable cost of C: VC.

We now have to adjust the response to the demand from the market to take into account the limit in capacity W.

So the profit function of C that has to be maximised depends upon the regions of probability space and can be written:

$$\pi_i(x_L,x_s \mid u,Q_s,\Omega i)=R_i(x_{Li}\mid u,\Omega i)+\min(W-x_L,x_s)P_s-VC(x_L+x_s) \qquad (1.26)$$

$$\text{s.t. :}\begin{cases} x_L+x_s\le W \\ 0\le x_L\le u \\ 0\le x_s\le Q_s \\ 0\le u\le Q_L \\ 0\le P_s \\ 0\le\theta_c\le c\le p_{La} \\ 0\le\theta_s\le c \\ 0\le q_{La}\le q_L \end{cases}$$

The decision regarding the demand received from the spot market Q_s depends upon the value of the spot price: if $P_s < VC$ then, C has no interest in allotting any capacity and $x_s = 0$. So when $P_s < VC$ then the profit function is written as follows:

$$\pi_i\left(x_L, x_s \mid u, Q_s, \Omega i\right) = R_i\left(x_{Li} \mid u, \Omega i\right) - x_L VC \tag{1.27}$$

By inspection, we see that the equations of the optimal allocation according to the regions of probability space reflect exactly the results established in 3.5.4.

We now consider the case when $P_s > VC$.

Case 1: Ω1

$\pi_1\left(x_L, x_s \mid u, Q_s, \Omega 1\right) = x_L\left(u\right)c + \left(q_L - x_L\left(u\right)\right)\theta_s + \min\left(W - x_L\left(u\right), x_s\right)P_s - VC\left(x_L\left(u\right) + x_s\right)$ We first optimize in x_L because this is the first decision to be taken by C:

$$x_L^* = u^* = Q_L$$
$$\pi_1\left(x_L^*, x_s \mid u^*, Q_s, \Omega 1\right) = Q_L c + \left(q_L - Q_L\right)\theta_s + \min\left(W - Q_L, x_s\right)P_s - VC\left(Q_L + x_s\right)$$

The optimization in x_s comes immediately:

$$x_s^* = Q_s$$
$$\pi_1\left(x_L^*, x_s^* \mid u^*, Q_s, \Omega 1\right) = Q_L c + \left(q_L - Q_L\right)\theta_s + \min\left(W - Q_L, Q_s\right)P_s - VC\left(Q_L + Q_s\right)$$

Much as in 3.5.4, we can write the other profit functions in the distinct regions of probability space. We will just recapitulate the results in the following table:

Ωi	u_i^*	x_{Li}^*	$\pi_{\Omega i}$	$C_{\Omega i}$
Ω1	Q_L	Q_L	$Q_L c+(q_L-Q_L)\theta_s+\min(W-Q_L,Q_s)P_s-VC(Q_L+Q_s)$	$Q_L c+(q_L-Q_L)\theta_s$
Ω2	q_L	q_L	$q_L c+(Q_L-q_L)(P_s+\theta_s)+\min(W-Q_L,Q_s)P_s-VC(Q_L+Q_s)$	$q_L c+(Q_L-q_L)(P_s+\theta_s)$
Ω3	Q_L	Q_L	$q_L c+(Q_L-q_L)p_{La}+\min(W-Q_L,Q_s)P_s-VC(Q_L+Q_s)$	$q_L c+(Q_L-q_L)p_{La}$
Ω4	Q_L	q_L	$q_L c+(Q_L-q_L)(P_s-\theta_c)+\min(W-Q_L,Q_s)P_s-VC(Q_L+Q_s)$	$q_L c+(Q_L-q_L)(P_s-\theta_c)$
Ω5	q_L	q_L	$q_L c+q_{La}\theta_s+(Q_L-q_L-q_{La}+\min(W-Q_L,Q_s)y)P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}\theta_s+(Q_L-q_L)P_s$
Ω6	Q_L	Q_L	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La}+\min(W-Q_L,Q_s))P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La})P_s$
Ω7	Q_L	q_L	$q_L c+q_{La}\theta_c+(Q_L-q_L+\min(W-Q_L,Q_s))P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}\theta_c+(Q_L-q_L)P_s$
Ω8	q_L	q_L	$q_L c+q_{La}\theta_s+(W-q_L-q_{La}+\min(W-Q_L,Q_s))P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}\theta_s+(Q_L-q_L)P_s$
Ω9	Q_L	Q_L	$q_L c+q_{La}p_{La}+(W-q_L-q_{La}+\min(W-Q_L,Q_s))P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}p_{La}+(Q_L-q_L-q_{La})P_s$
Ω10	Q_L	q_L	$q_L c+q_{La}\theta_c+(W-q_L+\min(W-Q_L,Q_s))P_s-VC(Q_L+Q_s)$	$q_L c+q_{La}\theta_c+(Q_L-q_L)P_s$

Table 2: Regions of probability space with relevant optimal decision and objective expression

3.5.6. *Expected cost and variance of transport cost*

Given that we now have defined the costs to the shipper over all regions of the probability space, we can define her expected cost as a function of the received demand Q_L and P_s using the notation introduced in 3.

$$E\left(C\left(u^*,x_L^*\right)\right)=\int_0^\infty\int_0^\infty C\left(u^*,x_L^*\right)f\left(Q_L,P_s\right)dQ_L dP_s \qquad (1.28)$$

When we open up this equation among the different regions we have:

$$E\left(C\left(u^*,x_L^*\right)\right)=\iint_{\Omega 1}\left(Q_L c+\left(q_L-Q_L\right)\theta_s\right)f\left(Q_L,P_s\right)dQ_L dP_s+$$
$$\iint_{\Omega 2}\left(q_L c-\left(Q_L-q_L\right)\left(P_s-\theta_s\right)\right)f\left(Q_L,P_s\right)dQ_L dP_s+$$
$$\iint_{\Omega 3}\left(q_L c+\left(Q_L-q_L\right)p_{La}\right)f\left(Q_L,P_s\right)dQ_L dP_s+$$
$$\iint_{\Omega 4}\left(q_L c+\left(Q_L-q_L\right)\left(P_s-\theta_c\right)\right)f\left(Q_L,P_s\right)dQ_L dP_s+ \qquad (1.29)$$
$$\iint_{\Omega 5\cup\Omega 8}\left(q_L c+\left(Q_L-q_L\right)\left(P_s-\theta_s\right)+\left(Q_L-q_L-q_{La}\right)P_s\right)f\left(Q_L,P_s\right)dQ_L dP_s+$$
$$\iint_{\Omega 6\cup\Omega 9}\left(q_L c+q_{La}p_{La}+\left(Q_L-q_L-q_{La}\right)P_s\right)f\left(Q_L,P_s\right)dQ_L dP+$$
$$\iint_{\Omega 7\cup\Omega 10}\left(q_L c+q_{La}\theta_c+\left(Q_L-q_L\right)P_s\right)f\left(Q_L,P_s\right)dQ_L dP_s$$

The regions 5 and 8, 6 and 9 and 7 and 10 have the same objective function (they only change for carrier C).

$$0\le u\le Q_L$$
$$C(u)=\begin{cases} cu+\left(q_L-u\right)\theta_s+\left(Q_L-u\right)P_s & : \quad 0\le u<q_L \\ cq_L+\left(u-q_L\right)p_{La}+\left(Q_L-u\right)P_s & : q_L\le u\le q_L+q_{La} \\ cq_L+q_{La}p_{La}+\left(u-q_L-q_{La}\right)P_s & : \quad q_L+q_{La}<u\le\min\left(W,Q_L\right)\end{cases} \qquad (1.30)$$

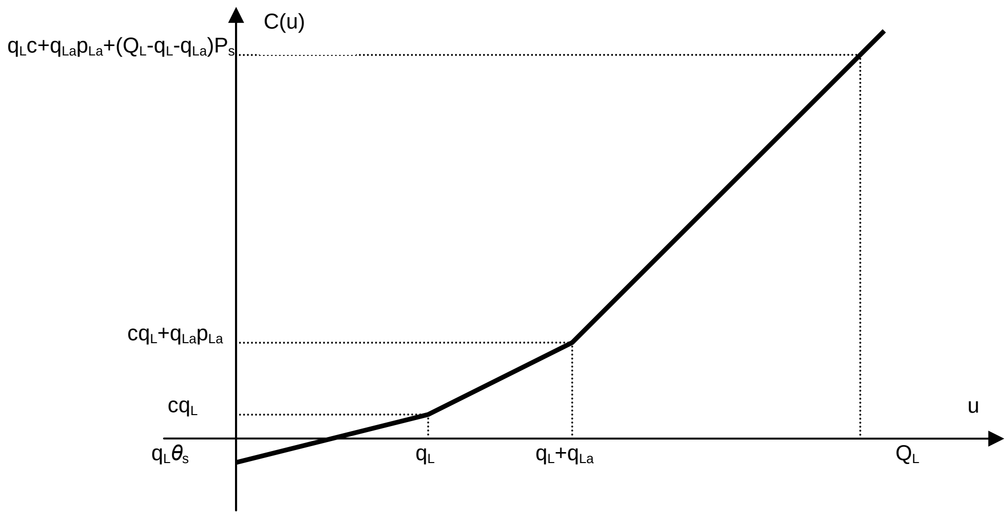

Fig.6: Behaviour of C(u)

5. INFORMATION SCENARIO ANALYSIS

We can now start modelling how each actor behaves according to the information he holds privately or that is common to both and see analytically the impact on the objective functions of C and S_L. Figure 2 in 3 above describes the chain of events in time.

In the first scenario, the information about the realized demands for the shippers is common knowledge to both shipper and the carrier, so is the spot market price for carrying that particular cargo at that particular period. In the second scenario, the capacity of C is unknown to S_L. In the third scenario, C's capacity is unknown to S_L and S_L's demand is unknown to C. In the fourth scenario, the carrier's capacity is known to S_L but S_L's demand is not known to the carrier.

We put a superscript index for each scenario on the carrier profit, shipper cost and standard deviation functions ($\pi_C^1; C^1; \sigma^1; R^1$ for scenario 1 for example).

3.1. *Scenario 1: Perfect information:*

The carrier and shipper share information truthfully, as if coordinated by a single centralized organization. According to the observed demands and spot price, shipper S_L decides to allocate the maximum of the realized demand to C and C allocates the maximum of his capacity to satisfy S_L.

$$u = Q_L,\ \ x_L = \min\left(W, Q_L\right) \quad (1.31)$$

So, their situation can be resumed below:

$Q_L \in \Omega 1$: realized demand is less than contracted capacity. C offers remaining capacity to the market according to revealed demand Qs. Any capacity of C left over is lost. C performs the transport service. Payout occurs; Shipper S_L pays a penalty to C for committed capacity unused.

$$C_{\Omega 1}^1\left(u^{1*} \mid x_L, \Omega 1\right) = Q_L c + \left(q_L - Q_L\right)\theta_s \quad (1.32)$$

$$R_{\Omega 1}^1\left({x_L}^{1*}, u^{1*} \mid \Omega 1\right) = Q_L c + \left(q_L - Q_L\right)\theta_s \quad (1.33)$$

$Q_L \in \Omega 2 \cup \Omega 3 \cup \Omega 4$: realized demand is more than contracted capacity but still within range of menu of prices included in the contract. Shipper buys additional capacity to C at the corresponding price in the contract menu. For this part of the demand received, S_L forfeits using the spot market, so no account is taken of the level of the spot versus p_{La} and the penalties. The excess capacity of C is offered to the market in accordance with market realized demand. Any remaining capacity of C is lost. Transport is performed and paid for. No penalties are due. The revenue and cost equations for S_L and C are the same as (1.12) in Case 3 above:

$$\begin{aligned} R_{\Omega 2\cup\Omega 3\cup\Omega 4}^1\left({x_L}^{1*}, u^{1*} \mid \Omega 2 \cup \Omega 3 \cup \Omega 4\right) &= q_L c + \left(Q_L - q_L\right) p_{La} \\ C_{\Omega 2\cup\Omega 3\cup\Omega 4}^1\left(u^{1*}, {x_L}^{1*} \mid \Omega 2 \cup \Omega 3 \cup \Omega 4\right) &= q_L c + \left(Q_L - q_L\right) p_{La} \end{aligned} \quad (1.34)$$

$Q_L \in \Omega 5 \cup \Omega 6 \cup \Omega 7$: realized demand exceeds not only contracted capacity but also the extra capacity of the menu of prices included in the ex-ante contract. Shipper S_L must complement the committed capacity of C by buying from C extra capacity at the going spot market price of that period. C's remaining capacity is offered to the market in the limit of Qs and W-Q_L. Payout occurs. The revenue and cost equations for S_L and C are:

$$\begin{aligned} R_{\Omega 5\cup\Omega 6\cup\Omega 7}^1\left({x_L}^{1*}, u^{1*} \mid \Omega 5 \cup \Omega 6 \cup \Omega 7\right) &= q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s \\ C_{\Omega 5\cup\Omega 6\cup\Omega 7}^1\left(u^{1*}, {x_L}^{1*} \mid \Omega 5 \cup \Omega 6 \cup \Omega 7\right) &= q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s \end{aligned} \quad (1.35)$$

$Q_L \in \Omega 8 \cup \Omega 9 \cup \Omega 10$: realized demand exceeds C's total capacity. S_L must go to the spot market for the excess demand at the going spot price. C cannot satisfy any of the demand received from the spot market.

$$\begin{aligned} R^1_{\Omega 8\cup\Omega 9\cup\Omega 10}\left(x_L^{1*}, u^{1*} \mid \Omega 5 \cup \Omega 6 \cup \Omega 7\right) &= q_L c + q_{La} p_{La} + \left(W - q_L - q_{La}\right) P_s \\ C^1_{\Omega 8\cup\Omega 9\cup\Omega 10}\left(u^{1*}, x_L^{1*} \mid \Omega 5 \cup \Omega 6 \cup \Omega 7\right) &= q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s \end{aligned} \tag{1.36}$$

The optimized revenue function R for C varies according to the different values of P_s and the sharing of capacity between S_L and the market:

$$\pi_c^1\left(x_L^{1*}, x_s^{1*}, Q_L, Q_s, P_s\right) - VC\left(x_L, x_s\right) = \begin{cases} Q_L c + \left(q_L - Q_L\right)\theta_s + \min\left(W - Q_L, x_s\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 1 \\ q_L c + \left(Q_L - q_L\right) p_{La} + \min\left(W - Q_L, x_s\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 2 \cup \Omega 3 \cup \Omega 4 \\ q_L c + q_{La} p_{La} + \left(\left(Q_L - q_L - q_{La}\right) + \min\left(W - Q_L, x_s\right)\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 5 \cup \Omega 6 \cup \Omega 7 \\ q_L c + q_{La} p_{La} + \left(W - q_L - q_{La}\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 8 \cup \Omega 9 \cup \Omega 10 \end{cases} \tag{1.37}$$

The optimized cost function of S_L becomes:

$$C^1\left(u^{1*}, x_L^{1*}, Q_L, P_s\right) = \begin{cases} Q_L c + \left(q_L - Q_L\right)\theta_s, \forall\left(Q_L, P_s\right) \in \Omega 1 \\ q_L c + \left(Q_L - q_L\right) p_{La}, \forall\left(Q_L, P_s\right) \in \Omega 2 \cup \Omega 3 \cup \Omega 4 \\ q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 5 \cup \Omega 6 \cup \Omega 7 \cup \Omega 8 \cup \Omega 9 \cup \Omega 10 \end{cases} \tag{1.38}$$

The optimized cost function of S_L becomes:

$$C^1\left(u^{1*}, x_L^{1*}, Q_L, P_s\right) = \begin{cases} Q_L c + \left(q_L - Q_L\right)\theta_s, \forall\left(Q_L, P_s\right) \in \Omega 1 \\ q_L c + \left(Q_L - q_L\right) p_{La}, \forall\left(Q_L, P_s\right) \in \Omega 2 \cup \Omega 3 \cup \Omega 4 \\ q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 5 \cup \Omega 6 \cup \Omega 7 \cup \Omega 8 \cup \Omega 9 \cup \Omega 10 \end{cases} \tag{1.39}$$

In this scenario, the Stackelberg position of S_L does not influence the outcome since no deviation will occur.

The conditional expected cost as a function of the received demand Q_L subject to P_s comes to:

$$\begin{aligned} E\left(C^1\left(u^{1*}, x_L^{1*}\right)\right) = &\int_{P_s \in \Omega 1} \int_{Q_L \in \Omega 1} \left(Q_L c + \left(q_L - Q_L\right)\theta_s\right) f\left(Q_L, P_s\right) dQ_L dP_s + \\ &\int_0^\infty \int_{Q_L \in \Omega 2 \cup \Omega 3 \cup \Omega 4} \left(q_L c + \left(Q_L - q_L\right) p_{La}\right) f\left(Q_L, P_s\right) dQ_L dP_s + \\ &\int_0^\infty \int_{\Omega 5 \cup \Omega 6 \cup \Omega 7 \cup \Omega 8 \cup \Omega 9 \cup \Omega 10} \left(Q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s\right) f\left(Q_L, P_s\right) dQ_L dP_s \end{aligned} \tag{1.40}$$

When applying the calculation method designed earlier, we find that this function is always equal to 0 whatever the parameters of the contract, when demand and price follow marginal exponential distribution functions and the joint distribution is an exponential bivariate. This is as should be since, with or without contract, profits are the same.

3.2. *Scenario 2: Asymmetric information:*

C has private information on W, the transport capacity. Ex ante, S_L has verified that C has at his disposal sufficient capacity to comply with q_L. She did not or could not verify the existence or size of the additional capacity S_L has to invest in to meet the commitments of the menu of prices (possible sub-contractors to C, extension of capacity in future, changes in other client demand patterns, etc are all possible reasons for such lack of observation).

So C has an opportunity to deviate when P_s is higher than $p_{La} + \theta_c$. If C deviates, the demand in excess of q_L by S_L has to be offered to the spot market. So the cost increases for S_L. C has been modelled to take that same amount from the spot market at the spot price so as to make it easier to compare performance and rent transfer between both players in the conclusions. The exact demand Q_L of S_L is here assumed observable by both S_L and C. The cost function of S_L and the revenue function of C have been established in Brusset 2004.

- $(Q_L, P_s) \in \Omega 1$

The same pattern as in scenario 1, no change in decisions or functions:

$$C^2_{\Omega 1}\left(x_L^{2*}, u^{2*}, P_s\right) = Q_L c + \left(q_L - Q_L\right)\theta_s \tag{1.41}$$

- $(Q_L, P_s) \in \Omega 2 \cup \Omega 3$

Here again the same pattern of behaviour is observed for both actors as in scenario 1

- $(Q_L, P_s) \in \Omega 4$

S_L wishes to allocate min(Q_L,q_L+q_{La}) to C. C refuses and decides to allocate all his capacity in excess of qL to the spot market, he pays a penalty to S_L. S_L turns to the spot market for $(Q_L - q_L)$, and receives the penalty from C. The cost and revenue functions are:

$$\begin{aligned} R^2_{\Omega 4}\left(x_L^{2*}, u^{2*} \mid \Omega 4\right) &= q_L c + \left(Q_L - q_L\right)\left(P_s - \theta_c\right) \\ C^2_{\Omega 4}\left(u^{2*}, x_L^{2*} \mid \Omega 4\right) &= q_L c + \left(Q_L - q_L\right)\left(P_s - \theta_c\right) \end{aligned} \tag{1.42}$$

$(Q_L, P_s) \in \Omega 5 \cup \Omega 6$

As in scenario 1, both stick to the contract and the equations of cost and revenue are:

$$\begin{aligned} R_{\Omega 5 \cup \Omega 6}\left(x_L^{2*}, u^{2*} \mid \Omega 5 \cup \Omega 6\right) &= q_L c + q_{La} p_{La} + \left(\min\left(W, Q_L\right) - q_L\right) P_s \\ C_{\Omega 5 \cup \Omega 6}\left(u^{2*}, x_L^{2*} \mid \Omega 5 \cup \Omega 6\right) &= q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s \end{aligned}$$

$(Q_L, P_s) \in \Omega 7$

C misbehaves when the spot is above the added menu of prices plus penalty for the portion of demand that is equal to q_{La}. The revenue and cost functions are:

$$\begin{aligned} R^2_{\Omega 7}\left(x_L^{2*}, u^{2*} \mid \Omega 7\right) &= q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s \\ C^2_{\Omega 7}\left(x_L^{2*}, u^{2*} \mid \Omega 7\right) &= q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s \end{aligned} \tag{1.43}$$

$(Q_L, P_s) \in \Omega 8 \cup \Omega 9$

Here demand exceeds overall carrier capacity W.

$$\begin{aligned} R_{\Omega 8 \cup \Omega 9}\left(x_L^{2*}, u^{2*} \mid \Omega 8 \cup \Omega 9\right) &= q_L c + q_{La} p_{La} + \left(W - q_L - q_{La}\right) P_s \\ C_{\Omega 8 \cup \Omega 9}\left(u^{2*}, x_L^{2*} \mid \Omega 8 \cup \Omega 9\right) &= q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s \end{aligned} \tag{1.44}$$

$\left(Q_L, P_s\right) \in \Omega 10$

Same behaviour as in Ω7, except that demand has exceeded W.

$$R^2_{\Omega 10}\left(x_L{}^{2*}, u^{2*} \mid \Omega 10\right) = q_L c + q_{La}\theta_c + \left(W - q_L\right) P_s$$
$$C^2_{\Omega 10}\left(x_L{}^{2*}, u^{2*} \mid \Omega 10\right) = q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s$$

This leads to the following cost function for S_L:

$$C^2\left(u^*, x_L{}^*, Q_L, P_s\right) = \begin{cases} Q_L c + \left(q_L - Q_L\right)\theta_s, \forall\left(Q_L, P_s\right) \in \Omega 1 \\ q_L c + \left(Q_L - q_L\right) p_{La}, \forall\left(Q_L, P_s\right) \in \Omega 2 \cup \Omega 3 \\ q_L c + \left(Q_L - q_L\right)\left(P_s - \theta_c\right), \forall\left(Q_L, P_s\right) \in \Omega 4 \\ q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 5 \cup \Omega 6 \cup \Omega 8 \cup \Omega 9 \\ q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 7 \cup \Omega 10 \end{cases} \quad (1.45)$$

$\left(Q_L, P_s\right) \in \Omega 10$

Same behaviour as in Ω7, except that demand has exceeded W.

$$R^2_{\Omega 10}\left(x_L{}^{2*}, u^{2*} \mid \Omega 10\right) = q_L c + q_{La}\theta_c + \left(W - q_L\right) P_s$$
$$C^2_{\Omega 10}\left(x_L{}^{2*}, u^{2*} \mid \Omega 10\right) = q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s$$

This leads to the following cost function for S_L:

$$C^2\left(u^*, x_L{}^*, Q_L, P_s\right) = \begin{cases} Q_L c + \left(q_L - Q_L\right)\theta_s, \forall\left(Q_L, P_s\right) \in \Omega 1 \\ q_L c + \left(Q_L - q_L\right) p_{La}, \forall\left(Q_L, P_s\right) \in \Omega 2 \cup \Omega 3 \\ q_L c + \left(Q_L - q_L\right)\left(P_s - \theta_c\right), \forall\left(Q_L, P_s\right) \in \Omega 4 \\ q_L c + q_{La} p_{La} + \left(Q_L - q_L - q_{La}\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 5 \cup \Omega 6 \cup \Omega 8 \cup \Omega 9 \\ q_L c + q_{La}\theta_c + \left(Q_L - q_L\right) P_s, \forall\left(Q_L, P_s\right) \in \Omega 7 \cup \Omega 10 \end{cases} \quad (1.46)$$

3.3. *Scenario 3: Private information:*

In this scenario, C has private information on W, S_L has private information on the demand Q_L: so both have an option to behave opportunistically according to the spot price P_s. Each sticks to q_L, basic capacity contracted for. In this last scenario, the menu of prices is unenforceable. For any spot price either higher or lower than the menu price p_{La} according to the additional capacity necessary, either the shipper or the carrier decides to go to the spot market. The other party, for lack of knowledge of capacity or cargo, cannot ask for nor receive any compensation.

- $\left(Q_L, P_s\right) \in \Omega 1$

  No deviation occurs, the functions are the same as in scenario 1.

- $\left(Q_L, P_s\right) \in \Omega 2$

S_L deviates and prefers to deal at the spot price and declares to C that, as there is no extra cargo to be taken, she has no penalty to pay. We have included in the model revenue to C for that cargo allocated to the spot market for comparison purposes, but the penalty that S_L would have paid had C known about that extra cargo is not included in the revenue. The revenue and cost functions differ from (1.10) in case 2 above:

$$\begin{aligned} R^3_{\Omega 2}\left(x_L{}^{3*}, u^{3*} \mid \Omega 2\right) &= q_L c + \left(Q_L - q_L\right) P_s \\ C^3_{\Omega 2}\left(u^{3*}, x_L{}^{3*} \mid \Omega 2\right) &= q_L c + \left(Q_L - q_L\right) P_s \end{aligned} \tag{1.47}$$

- $\left(Q_L, P_s\right) \in \Omega 3$

  The penalties are sufficient to dissuade both shipper and carrier to deviate in this region of probability space, so the revenue and cost functions for the optimal allocations are the same as in scenario 1.

- $\left(Q_L, P_s\right) \in \Omega 4$

  The carrier deviates as in scenario 2 and allocates his capacity to the market, to the detriment of S_L. However, no penalty is paid, hence, the cost and revenue functions change to:

$$\begin{aligned} R^3_{\Omega 4}\left(x_L{}^{3*}, u^{3*} \mid \Omega 4\right) &= q_L c + \left(Q_L - q_L\right) P_s \\ C^3_{\Omega 4}\left(u^{3*}, x_L{}^{3*} \mid \Omega 4\right) &= q_L c + \left(Q_L - q_L\right) P_s \end{aligned} \tag{1.48}$$

- $\left(Q_L, P_s\right) \in \Omega 5$

  S_L reports less demand to C, completes for the rest from the spot market. C has no way to check, so does not ask for, nor receive, the corresponding penalty. The cost and revenue functions are now written:

$$\begin{aligned} R^3_{\Omega 5}\left(x_L{}^{3*}, u^{3*} \mid \Omega 5\right) &= q_L c + \left(Q_L - q_L\right) P_s \\ C^3_{\Omega 5}\left(u^{3*}, x_L{}^{3*} \mid \Omega 5\right) &= q_L c + \left(Q_L - q_L\right) P_s \end{aligned} \tag{1.49}$$

- $\left(Q_L, P_s\right) \in \Omega 6$

  As in scenario1, no deviation takes place. We have the same revenue and cost functions.

- $\left(Q_L, P_s\right) \in \Omega 7$

  As in scenario 2, C refuses to carry any cargo above q_L and does not pay any penalty either, arguing that he lacks capacity (unobservable by S_L):

$$\begin{aligned} R^3_{\Omega 7}\left(x_L{}^{3*}, u^{3*} \mid \Omega 7\right) &= q_L c + \left(Q_L - q_L\right) P_s \\ C^3_{\Omega 7}\left(u^{3*}, x_L{}^{3*} \mid \Omega 7\right) &= q_L c + \left(Q_L - q_L\right) P_s \end{aligned} \tag{1.50}$$

- $\left(Q_L, P_s\right) \in \Omega 8$

  Same as in Ω5 except that demand addressed to S_L exceeds C's total capacity W.

$$R^3_{\Omega 8}\left(x_L{}^{3*},u^{3*}\,|\,\Omega 8\right)=q_L c+\left(W-q_L\right)P_s$$
$$C^3_{\Omega 8}\left(u^{3*},x_L{}^{3*}\,|\,\Omega 8\right)=q_L c+\left(Q_L-q_L\right)P_s \tag{1.51}$$

- $\left(Q_L,P_s\right)\in\Omega 9$

  Same as in Ω9 of scenario 1.

$$R^3_{\Omega 10}\left(x_L{}^{3*},u^{3*}\,|\,\Omega 10\right)=q_L c+\left(W-q_L\right)P_s$$
$$C^3_{\Omega 10}\left(u^{3*},x_L{}^{3*}\,|\,\Omega 10\right)=q_L c+\left(Q_L-q_L\right)P_s \tag{1.52}$$

- $\left(Q_L,P_s\right)\in\Omega 9$

  Same as in Ω9 of scenario 1.

- $\left(Q_L,P_s\right)\in\Omega 10$

  In difference with Ω7, W as a limit kicks in for the carrier:

$$R^3_{\Omega 10}\left(x_L{}^{3*},u^{3*}\,|\,\Omega 10\right)=q_L c+\left(W-q_L\right)P_s$$
$$C^3_{\Omega 10}\left(u^{3*},x_L{}^{3*}\,|\,\Omega 10\right)=q_L c+\left(Q_L-q_L\right)P_s \tag{1.53}$$

The cost and revenue functions are the ones previously mentioned but with the extra portion of realized demand exceeding contractual obligations that in any case will be allocated to C at spot market price.

The optimal cost function for S_L is now reduced to:

$$C^3\left(u^{3*},x_L{}^{3*},Q_L,P_s\right)=\begin{cases}Q_L c+\left(q_L-Q_L\right)\theta_s, & \forall\left(Q_L,P_s\right)\in\Omega 1\\ q_L c+\left(Q_L-q_L\right)P_s, & \forall\left(Q_L,P_s\right)\in\Omega 2\cup\Omega 4\cup\Omega 5\cup\Omega 7\cup\Omega 8\cup\Omega 10\\ q_L c+q_{La}p_{La}+\left(Q_L-q_L-q_{La}\right)P_s, & \forall\left(Q_L,P_s\right)\in\Omega 6\cup\Omega 9\\ q_L c+\left(Q_L-q_L\right)p_{La}, & \forall\left(Q_L,P_s\right)\in\Omega 3\end{cases} \tag{1.54}$$

3.4. *Scenario 4: Private information:*

In this scenario, C has private information on W, S_L has private information on the

3.6. *Comparison between scenarios*

3.6.1. *Comparison between scenario 1 and 2*

The differences between scenario 1 and scenario 2 can be calculated by using the partitions already created (see figure 3): we have a difference only when Q_L comes in between q_L and q_L+q_{La} and the spot price happens to be above p_{La}.+ θ_c, which belongs to partition $\Omega 4\cup\Omega 7\cup\Omega 10$.

$\forall\left(Q_L,P_s\right)\in\Omega 4\cup\Omega 7\cup\Omega 10$

$$\pi^2\left(x_L{}^{2*},u^{2*},Q_L,P_s\right)-\pi^1\left(x_L{}^{1*},u^{1*},Q_L,P_s\right)=\left(Q_L-q_L\right)\left(P_s-\theta_c-p_{La}\right) \tag{1.55}$$

$$C^2\left(u^{2*},x_L{}^{2*},Q_L,P_s\right)-C^1\left(u^{1*},x_L{}^{1*},Q_L,P_s\right)=\left(Q_L-q_L\right)\left(P_s-\theta_c-p_{La}\right) \tag{1.56}$$

By definition of the contract parameters, we can write:

$\forall (Q_L, P_s) \in \Omega 4 \cup \Omega 7 \cup \Omega 10$

$$\begin{aligned} &\pi^2\left(x_L{}^{2*}, u^{2*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right) \geq 0 \\ &C^2\left(u^{2*}, x_L{}^{2*}, Q_L, P_s\right) - C^1\left(u^{1*}, x_L{}^{1*}, Q_L, P_s\right) \geq 0 \end{aligned} \quad (1.57)$$

Both results are positive if there is but one instance of both the spot price higher than the menu of prices fixed in the contract plus the carrier penalty and existence of cargo to be taken in excess of base commitment q_L.

There is a transfer of resources from S_L to C when C can deviates from truthful behaviour by hiding the exact capacity he has at his disposal and withhold extra capacity from S_L to sell it to the spot market at a higher price.

The conditional expected cost of the difference in information is written:

$$\begin{aligned} E\left(C^2\left(u^{2*}, x_L{}^{2*} \mid Q_L, P_s\right) - C^1\left(u^{1*}, x_L{}^{1*} \mid Q_L, P_s\right)\right) &= \iint_{\Omega 4 \cup \Omega 7 \cup \Omega 10} \left(\left(Q_L - q_L\right)\left(P_s - p_{La}\right)\right) f\left(Q_L, P_s\right) dQ_L dP_s \\ &= E\left(\pi^2\left(x_L{}^{2*}, u^{2*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right)\right) \end{aligned} \quad (1.58)$$

The variance of the transport cost to S_L increases with the variances of the component laws: ζ_L and P_s affected by the values given to the contractual parameters.

3.6.2. *Comparison between scenario 1 and 3*

The differences occur in regions Ω2,Ω4,Ω5, Ω7, Ω8 and Ω10 when either the shipper or the carrier has an incentive to deviate. By investigation, these come to:

$\forall (Q_L, P_s) \in \Omega 2$

$$\pi^3\left(x_L{}^{3*}, u^{3*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right) = \left(Q_L - q_L\right)\left(p_{La} - P_s\right) \quad (1.59)$$

$$C^3\left(x_L{}^{3*}, u^{3*}, Q_L, P_s\right) - C^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right) = \left(Q_L - q_L\right)\left(p_{La} - P_s\right) \quad (1.60)$$

$$C^3\left(x_L{}^{3*}, u^{3*}, Q_L, P_s\right) - C^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right) = \left(Q_L - q_L\right)\left(p_{La} - P_s\right) \quad (1.61)$$

$\forall (Q_L, P_s) \in \Omega 4$

$$\pi^3\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) = \left(Q_L - q_L\right)\left(P_s - p_{La}\right) \quad (1.62)$$

$$C^3\left(x_L{}^{3*}, u^{3*}, Q_L, P_s\right) - C^1\left(x_L{}^{1*}, u^{1*}, Q_L, P_s\right) = \left(Q_L - q_L\right)\left(P_s - p_{La}\right) \quad (1.63)$$

$\forall (Q_L, P_s) \in \Omega 5 \cup \Omega 8$

$$\pi^3\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) = q_{La}\left(P_s - p_{La}\right) \quad (1.64)$$

$\forall (Q_L, P_s) \in \Omega 7 \cup \Omega 10$

$$\pi^3\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) = q_{La}\left(P_s - p_{La}\right) \quad (1.65)$$

$\forall (Q_L, P_s) \in \Omega 1 \cup \Omega 3 \cup \Omega 6 \cup \Omega 9$

$$\pi^3\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) - \pi^1\left(x_L{}^{*}, u^{*}, Q_L, P_s\right) = 0 \quad (1.66)$$

$$C^3\left(x_L^{\ *},u^*,Q_L,P_s\right)-C^1\left(x_L^{\ *},u^*,Q_L,P_s\right)=0 \tag{1.67}$$

$$C^3\left(x_L^{\ *},u^*,Q_L,P_s\right)-C^1\left(x_L^{\ *},u^*,Q_L,P_s\right)=0 \tag{1.68}$$

The conditional expectation of this difference subject to P_s and Q_L can be written as:

$$\begin{aligned}E\left(C^3\left(x_L^{3*},u^{3*}\right)-C^1\left(x_L^{1*},u^{1*}\right)\mid Q_L,P_s\right)=&\int_{P_s\in\Omega 2}\int_{Q_L\in\Omega 2}\left(\left(Q_L-q_L\right)\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{P_s\in\Omega 4}\int_{Q_L\in\Omega 4}\left(\left(Q_L-q_L\right)\left(P_s-p_{La}\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{P_s\in\Omega 5\cup\Omega 8}\int_{Q_L\in\Omega 5\cup\Omega 8}\left(q_{La}\left(P_s-p_{La}\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{P_s\in\Omega 7\cup\Omega 10}\int_{Q_L\in\Omega 7\cup\Omega 10}\left(q_{La}\left(P_s-p_{La}\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s\end{aligned} \tag{1.69}$$

Following the same reasoning, we can write the conditional expectation of the difference, subject to P_s and Q_L, of the profit to the carrier as:

$$\begin{aligned}E\left(\pi^3\left(x_L^{3*},u^{3*}\right)-\pi^1\left(x_L^{1*},u^{1*}\right)\mid Q_L,P_s\right)=&\int_{\Omega 2}\int_{\Omega 2}\left(\left(Q_L-q_L\right)\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 4}\int_{\Omega 4}\left(\left(Q_L-q_L\right)\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 5\cup\Omega 8}\int_{\Omega 5\cup\Omega 8}\left(q_{La}\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 7\cup\Omega 10}\int_{\Omega 7\cup\Omega 10}\left(q_{La}\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\end{aligned} \tag{1.70}$$

Following the same reasoning, we can write the conditional expectation of the difference, subject to P_s and Q_L, of the profit to the carrier as:

$$\begin{aligned}E\left(\pi^3\left(x_L^{3*},u^{3*}\right)-\pi^1\left(x_L^{1*},u^{1*}\right)\mid Q_L,P_s\right)=&\int_{\Omega 2}\int_{\Omega 2}\left(\left(Q_L-q_L\right)\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 4}\int_{\Omega 4}\left(\left(Q_L-q_L\right)\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 5\cup\Omega 8}\int_{\Omega 5\cup\Omega 8}\left(q_{La}\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\\&\int_{\Omega 7\cup\Omega 10}\int_{\Omega 7\cup\Omega 10}\left(q_{La}\left(p_{La}-P_s\right)\right)f\left(Q_L,P_s\right)dQ_LdP_s+\end{aligned} \tag{1.71}$$

These indications give guidance to the way the contractual parameters have to be negotiated by the shipper and the carrier so that if the information conditions are not given, at least the differences between both scenarios can be minimized. Such uncertainties and optimization of the contractual parameters will be the subject of another paper. For now, we have found that applying the preceding reasoning through a numerical study would give some indications as to the importance of the different parameters on behaviour by S_L and C.

So that we can have a feeling for the actual results, we study an exponential cumulative distribution function as an instance of the above continuous distribution.

3.6.3. *Instance using an exponential distribution*

3.6.3.1. *Description of bivariate exponential distribution*

We have chosen Downton's bivariate exponential distribution as described in Kotz et al.(2000) with the joint density function (pdf):

$$f(q,p,\lambda_1,\lambda_2,\rho)=\frac{\lambda_1\lambda_2}{1-\rho}\exp\left(-\frac{\lambda_1 q+\lambda_2 p}{1-\rho}\right)I_0\left(\frac{2(\rho\lambda_1\lambda_2 qp)^{1/2}}{1-\rho}\right) \qquad (1.72)$$

where, to simplify, $q = Q_L$, $p = P_s$, λ_1 and $\lambda_2 > 0$ and $I_0(z)=\sum_{k=0}^{\infty}(z/2)^{2k}/k!^2$ is the modified Bessel function of the first kind of order zero.

Let it be clear that here we limit our consideration to the case where the **correlation coefficient is positive or null**: ***0 < ρ < 1.*** We will restrict our study to the cases where spot market prices for freight transport and demands addressed to the shipper are **positively correlated**.

The above density was initially derived by Moran (1967). The marginal distributions of both Q and P are exponential with means 1/λ1 and 1/λ2 respectively. Since I0(0)=1, it is clear that Q and P are independent if and only if $\rho = 0$. Downton (1970) showed that ρ is the correlation coefficient of the two variates.

The marginal probability density functions can be written:

$$\begin{aligned} f_L(q) &= \lambda_1 e^{-\lambda_1 q} \\ f_s(p) &= \lambda_2 e^{-\lambda_2 p} \end{aligned} \qquad (1.73)$$

We can write the marginal distribution functions as:

$$\begin{aligned} F_1(q) &= \int_0^q \lambda_1 e^{-\lambda_1 t}dt = 1-e^{-\lambda_1 q} \\ F_2(p) &= \int_0^p \lambda_2 e^{-\lambda_2 t}dt = 1-e^{-\lambda_2 p} \end{aligned} \qquad (1.74)$$

So as not to bother the reader with tedious mathematical detail, the remainder of the demonstration is relegated to Annex I.

3.6.4. *Optimal contract parameters to minimize moral hazard on the carrier's part*

We will look for the optimal contract parameters so as to minimize the cost to the shipper of hidden action on the carrier's part in scenario 2.

We have to minimize (1.58) in 3.6.1, the difference between cost when both actors fully trust each other and information is available to all and the case when the carrier hides the available fleet information from the shipper:

$E\left(C^2\left(u^{2*},x_L^{2*},Q_L,P_s\right)-C^1\left(u^{1*},x_L^{1*},Q_L,P_s\right)\right)=\iint_{\Omega 4\cup\Omega 7\cup\Omega 10}\left((Q_L-q_L)(P_s-p_{La})\right)f(Q_L,P_s)\,dQ_L dP_s$ unifying the notation:

$$E\left(C^2\left(u^{*2},x_L^{*2},q,p,\right)-C^1\left(u^{*1},x_L^{*1},q,p\right)\right)=\iint_{\Omega 4\cup\Omega 7\cup\Omega 10}\left(\left(q-q_L\right)\left(p-p_{La}\right)\right)f\left(q,p\right)dqdp$$

Let us call EC_{2-1} this expected cost to be minimized. The segments of the probability regions which q, p describe are:

$q\in[q_L,\infty)$ and $p\in[p_{La}+\theta_c,\infty)$. Expanding, we can write:

$$EC_{2-1}=\iint_{\Omega 4\cup\Omega 7\cup\Omega 10} qpf\left(q,p\right)-q_L pf\left(q,p\right)-p_{La}qf\left(q,p\right)+q_L p_{La}f\left(q,p\right)dqdp \quad (1.75)$$

We can solve for the optimal parameters of the contract involved q_L, p_{La}, θ_c.

As per our notation in Annex I, this means that we have to minimize $A_4+A_7+A_{10}$.

We refer the reader to Annex II for the calculations.

For that, let us consider a numerical example for the marginal pdf of p, the spot market price for transport capacity and the demand addressed to the shipper q.

$f_L\left(q\right)=\lambda_1 e^{-\lambda_1 q}$ with average equal to100 ($\lambda_1=\frac{1}{100}$) and $f_s\left(p\right)=\lambda_2 e^{-\lambda_2 p}$ with average price equal to 5 ($\lambda_2=\frac{1}{5}$) and the correlation coefficient $\rho = 0.6$.

We now have to solve for only three parameters: θ_c, q_L and p_{La}.

Let us study these between minima and maxima expressed as functions of the mean and standard deviation in each pdf.

$$\frac{1}{\lambda_1}-\sqrt{\frac{1}{\lambda_1^2}}\le q_L\le\frac{1}{\lambda_1}+\sqrt{\frac{1}{\lambda_1^2}}\Rightarrow 0\le q_L\le 200$$

$$\frac{1}{\lambda_2}-\sqrt{\frac{1}{\lambda_2^2}}\le p_{La}\le\frac{1}{\lambda_2}+\sqrt{\frac{1}{\lambda_2^2}}\Rightarrow 0\le p_{La}\le 10$$

$$\frac{1}{2\lambda_2}-\frac{1}{2}\sqrt{\frac{1}{\lambda_2^2}}\le\theta_c\le\frac{1}{2\lambda_2}+\frac{1}{2}\sqrt{\frac{1}{\lambda_2^2}}\Rightarrow 0\le\theta_c\le 100,$$

where the penalty will be limited to half of the average spot price plus or minus one standard deviation of the spot price distribution. We arbitrarily consider that higher penalties would be too dissuasive for the carrier or might entail equal demand from the carrier regarding the reciprocal penalty for failing to receive demand from the shipper equal to the contracted capacity q_L.

The following graphs give an idea of the optimal combination of parameters to minimize the difference between the expected cost to the shipper when he is unable to verify true carrier capacity and respect of the transport contract .

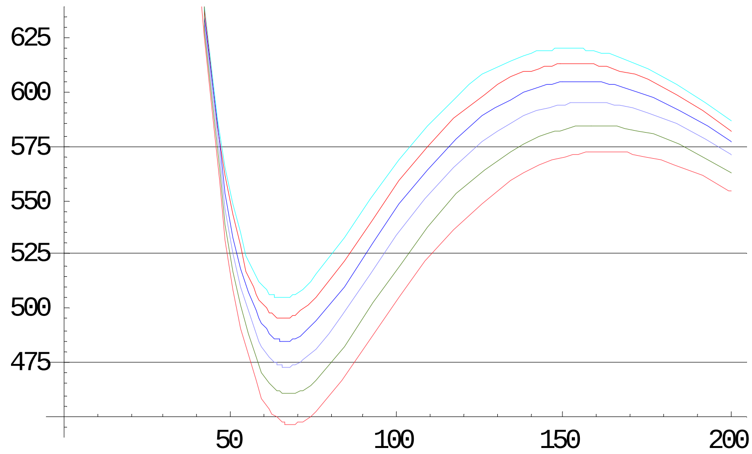

Fig. 7: Difference between expected cost in scenario 2 and 1, x axis represents q_L . 5 curves represent integer values from 5 to 10 for p_{La}

We see that the all curves dominate each other as the value of p_{La} increases. The higher value in p_{La} gives the lowest expected cost difference. We also can see that as pLa increases, the lowest point in cost is achieved at higher values for q_L. The shipper thus can strike a balance between the additional capacity price p_{La} and the spot price to align the carrier's behaviour.

This analysis can be extended to variations in the penalty parameter versus q_L. We could also represent 3D graphs combining two parameters at a time. We have not been able, however, to find a truly minimal expected cost value because of the extreme complexity and length of the cost function.

6. CONCLUSION

There is a lack of research and practice in approaching transport as a full fledged member of a supply chain. In this paper, we present transport as an individualized supply chain member with proper characteristics. We have modelled the impact and influence that information sharing and coordination with transport suppliers has on the efficiency of the supply chain. We have established that:

1. The contract in the mixed strategy must include a fixed capacity commitment and some additional flexibility in capacity (QF clause).
2. Penalties should be included: the carrier is penalised when he cannot comply with his contractual capacity engagements; the shipper is penalised when he cannot fulfil his buying engagements. We have shown that this ensures coordination.
3. The information imbalances induced by keeping private information as to the real transport capacity by the carrier, the real demand received by the shipper is detrimental to the overall efficiency of the supply chain, because it encourages deviant attitudes both from the carrier and the shipper.
4. Carefully crafted ex-ante contractual arrangements can substantially correct this information asymmetry. These contractual arrangements depend only on demand and price distribution characteristics.

One prolongation of the present paper will deal with optimizing contract parameters to reduce demand and price variability over all probability space.

The aim of the supply chain manager should be to reduce standard deviation because added cost standard deviation is an incentive, in a multi-period game, to increase margins at both levels of the supply chain increasing the notorious double margining phenomenon. The shipper increases his budgeted costs because he cannot ensure regularity of his cost and hence must protect himself by padding his transport budget; the carrier increases price of services because he has to contend with fixed cost non-scalable capacity and so must also preserve his financial health by higher than warranted profit margins.

Annex I

The expected cost function for the shipper can be rewritten as:

$$E\left(C\left(u^*, x_L^*\right)\right) = \sum_{i=1}^{7} \iint_{\Omega i} C_{\Omega i}\left(q, p\right) f\left(q, p_s\right) dqdp \qquad (2.1)$$

Let us call A_i each term of the sum:

$$E\left(C\left(u^*, x_L^*\right)\right) = \sum_{i=1}^{7} A_i\left(Q_L, P_s\right), \qquad \text{where} \quad \forall i, A_i = \iint_{\Omega i} C_{\Omega i}\left(q, p\right) f\left(q, p\right) dqdp \quad (2.2)$$

In Iliopoulos (2003), we find a very practical representation of Downton's bivariate density function as an expansion of a series. The pdf can be written as a series of products of a geometric probability mass function $\Psi\left(k, \rho\right) = \left(1-\rho\right)\rho^k; k = 0,1,2,...$ and of two independent Gamma random variables. The gamma variates have the shape parameters: k+1 and the scale parameters $\left(1-\rho\right)/\lambda_i, i \in \{1,2\}$. Their general form is:

$$g_k\left(x, \beta\right) = \frac{\beta}{\Gamma\left(k\right)}\left(\beta x\right)^{k-1} e^{\left(-\beta x\right)} \qquad (2.3)$$

where $\Gamma\left(k\right)$ is the gamma function of k which is defined by: $\Gamma\left(x\right) = \int_0^{\infty} s^{x-1} e^{-s} ds$, β is the shape parameter and k is the scale parameter as defined in Iliopoulos (2003). Applying our definition of β and k to (21), we have:

$$g_{k+1}\left(q, \frac{1-\rho}{\lambda_2}\right) = \frac{\lambda_2}{\left(1-\rho\right)\Gamma\left(k+1\right)}\left(\frac{q\lambda_2}{1-\rho}\right)^k e^{\left(\frac{q\lambda_2}{\rho-1}\right)} \qquad (2.4)$$

1.1. *Using results from Iliopoulos 2003*

As in Iliopoulos (2003), let K be a random variable having the above geometric distribution. Then **conditionally** on *K* = *k*, *Q* and *P* are independent gamma variates, which enable us to write:

$$f\left(x, y, \lambda_1, \lambda_2, \rho\right) = \sum_{k=0}^{\infty} \Psi\left(k, \rho\right) g_{k+1}\left(x, \frac{1-\rho}{\lambda_1}\right) g_{k+1}\left(y, \frac{1-\rho}{\lambda_2}\right) \qquad (2.5)$$

Where $g_{k+1}\left(x, \frac{1-\rho}{\lambda i}\right), i \in \{1,2\}$ are the pdf of the gamma random variables.

So, plugging into our formula:

$$A_i = \iint_{\Omega i} C_{\Omega i}\left(q, p\right) \sum_{k=0}^{\infty} \psi\left(k, \rho\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) dqdp \qquad (2.6)$$

Because of the linearity of integrals, we can write:

$$A_i = \sum_{k=0}^{\infty} \psi\left(k, \rho\right) \iint_{\Omega i} C_{\Omega i}\left(q, p\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) dqdp \qquad (2.7)$$

Moreover, we have defined the $C_{\Omega i}$ over the different probability regions. In each case of *i*, we can write the cost function in the following way:

$\forall i, i \in \{1,10\}, C_{\Omega i}(q,p) = \alpha_i p + \beta_i q + \gamma_i pq + \delta_i$

So we can write:

$$A_i = \sum_{k=0}^{\infty} \psi(k,\rho) \left[\begin{array}{l} \iint_{\Omega i} \alpha_i p g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dqdp + \\ \iint_{\Omega i} \beta_i q g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dqdp + \\ \iint_{\Omega i} \gamma_i pq g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dqdp + \\ \iint_{\Omega i} \delta_i g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dqdp \end{array} \right] \quad (2.8)$$

1.2. *Separating integrals*

We can here evaluate the integrals separately. We deal with the innermost integral first. And since both gamma variates g are independent, each is only expressed using just one variable. So, in the inner integral, the gamma variate on p is a constant, we can "take" it out of that integral. We have to specify segments over which p and q will range.

Let *Pi* and *Qi* be the *i*th segments to which *p* and *q* belong respectively and defining the region Ωi.

$$A_i = \sum_{k=0}^{\infty} \psi(k,\rho) \left[\begin{array}{l} \int_{p\in Pi} \alpha_i p g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) \left(\int_{q\in Qi} g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dq \right) dp + \\ \int_{p\in Pi} \beta_i g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) \left(\int_{q\in Qi} q g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dq \right) dp + \\ \int_{p\in Pi} \gamma_i p g_{k+1}\left(p, \frac{1-\rho}{\lambda_2}\right) \left(\int_{q\in Qi} q g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dq \right) dp + \\ \int_{p\in Pi} \delta_i g_{k+1}\left(p, \frac{1-\rho}{\lambda_1}\right) \left(\int_{q\in Qi} g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dq \right) dp \end{array} \right] \quad (2.9)$$

Usually, the distribution function of the Gamma pdf is not a closed form unless the shape parameter is an integer, which is our case. So we can integrate in parts the innermost integral in each term. Moreover, because the shape parameter (*k* in our case) is an integer, the distribution function of a Gamma pdf is by definition:

$$G_{k+1}^1(q,\lambda_1) = \int g_{k+1}\left(q, \frac{1-\rho}{\lambda_1}\right) dq = -\frac{\Gamma\left(k+1, \frac{q\lambda_1}{1-\rho}\right)}{\Gamma(k+1)} \quad (2.10)$$

where the $\Gamma\left(k+1, \frac{q\lambda_1}{1-\rho}\right)$ is the incomplete Gamma function of k and $q\frac{\lambda_1}{1-\rho}$. This function is written as:

$$\Gamma\left(k+1,\frac{q\lambda_1}{1-\rho}\right)=\int_{q\lambda_1/1-\rho}^{\infty} t^{k-1}e^{-t}dt \quad (2.11)$$

Let us call G_k^2 the following indefinite integral:

$$\begin{aligned} G_{k+1}^2(q,\lambda_1) &= \int q g_{k+1}\left(q,\frac{1-\rho}{\lambda_1}\right)dq \\ &= -\frac{1-\rho}{\lambda_1}\frac{\Gamma\left(k+2,\frac{q\lambda_1}{1-\rho}\right)}{\Gamma(k+1)} \end{aligned} \quad (2.12)$$

Now, we can replace for each occurrence in the double integrals:

$$A_i=\sum_{k=0}^{\infty}\psi(k,\rho)\begin{bmatrix} \int_{Pi}\alpha_i p g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)\left[G_{k+1}^2(q,\lambda_1)\right]_{Qi}dp+ \\ \int_{Pi}\beta_i g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)\left[G_{k+1}^1(q,\lambda_1)\right]_{Qi}dp+ \\ \int_{Pi}\gamma_i p g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)\left[G_{k+1}^2(q,\lambda_1)\right]_{Qi}dp+ \\ \int_{Pi}\delta_i g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)\left[G_{k+1}^1(q,\lambda_1)\right]_{Qi}dp \end{bmatrix} \quad (2.13)$$

We thus have an expression which depends only on the contract characteristics and the marginal distribution parameters of the variables *q* and *p*.

The expression in each remaining integral is a constant regarding the *p* variable. So we can "take out" the whole crotchet term of the integral:

$$A_i=\sum_{k=0}^{\infty}\psi(k,\rho)\begin{bmatrix} \alpha_i\left(\left[G_{k+1}^1(q,\lambda_1)\right]_{Qi}\right)\int_{Pi}p g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)dp+ \\ \beta_i\left(\left[G_{k+1}^2(q,\lambda_1)\right]_{Qi}\right)\int_{Pi}g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)dp+ \\ \gamma_i\left(\left[G_{k+1}^2(q,\lambda_1)\right]_{Qi}\right)\int_{Pi}p g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)dp+ \\ \delta_i\left(\left[G_{k+1}^1(q,\lambda_1)\right]_{Qi}\right)\int_{Pi}g_{k+1}\left(p,\frac{1-\rho}{\lambda_2}\right)dp \end{bmatrix} \quad (2.14)$$

We have already solved the integral of the $g_k(.)$ function and the product of that function by *p*, so we write directly:

$$A_i = \sum_{k=0}^{\infty} \psi(k,\rho) \begin{bmatrix} \alpha_i \left(\left[G_{k+1}^1(q,\lambda_1) \right]_{Qi} \right) \left(\left[G_{k+1}^2(p,\lambda_2) \right]_{Pi} \right) + \\ \beta_i \left(\left[G_{k+1}^2(q,\lambda_1) \right]_{Qi} \right) \left(\left[G_{k+1}^1(p,\lambda_2) \right]_{Pi} \right) + \\ \gamma_i \left(\left[G_{k+1}^2(q,\lambda_1) \right]_{Qi} \right) \left(\left[G_{k+1}^2(p,\lambda_2) \right]_{Pi} \right) + \\ \delta_i \left(\left[G_{k+1}^1(q,\lambda_1) \right]_{Qi} \right) \left(\left[G_{k+1}^1(p,\lambda_2) \right]_{Pi} \right) \end{bmatrix} \quad (2.15)$$

1.3. *Rearranging terms*

We first extract the coefficients:

$$\begin{aligned} A_i = \alpha_i & \left(\sum_{k=0}^{\infty} \psi(k,\rho) \left[G_{k+1}^1(q,\lambda_1) \right]_{Qi} \left[G_{k+1}^2(p,\lambda_2) \right]_{Pi} \right) + \\ \beta_i & \left(\sum_{k=0}^{\infty} \psi(k,\rho) \left[G_{k+1}^2(q,\lambda_1) \right]_{Qi} \left[G_{k+1}^1(p,\lambda_2) \right]_{Pi} \right) + \\ \gamma_i & \left(\sum_{k=0}^{\infty} \psi(k,\rho) \left[G_{k+1}^2(q,\lambda_1) \right]_{Qi} \left[G_{k+1}^2(p,\lambda_2) \right]_{Pi} \right) + \\ \delta_i & \left(\sum_{k=0}^{\infty} \psi(k,\rho) \left[G_{k+1}^1(q,\lambda_1) \right]_{Qi} \left[G_{k+1}^1(p,\lambda_2) \right]_{Pi} \right) \end{aligned} \quad (2.16)$$

One can see that, once all the terms are expanded within the sum signs, the right hand side (rhs) becomes a sum over k of terms which can best be expressed as products of normalized gamma functions. These products are of the form:

$$\frac{\Gamma(k+1+m,x)}{\Gamma(k+1)} \frac{\Gamma(k+1+l,y)}{\Gamma(k+1)} \quad (2.17),$$

with x and y taking the corresponding values of the borders of each segment on p and q, $k=0, 1, 2...$ to infinite, $m,l \in \{0,1,2\}$. In all, there are, within the sum term, sixteen products of this kind (two functions times 4 factors times two limits). So by externalising the factors independent from k, the lhs can be written as a sum over sixteen terms with eight term expressed in this way:

$$u_i \sum_{k=0}^{\infty} \psi(k,\rho) \frac{\Gamma(k+1,x)}{\Gamma(k+1)} \frac{\Gamma(k+1,y)}{\Gamma(k+1)} \quad (2.18),$$

where the u_i term ($i \in \{1,8\}$) is the result of the product of the independent factors affecting this particular sum over k.

Of the remaining eight, four are of the form:

$$v_i \sum_{k=0}^{\infty} \psi(k,\rho) \frac{\Gamma(k+2,x)}{\Gamma(k+1)} \frac{\Gamma(k+1,y)}{\Gamma(k+1)} \quad (2.19)$$

and the last four are written in the following form:

$$w_i \sum_{k=0}^{\infty} \psi(k,\rho) \frac{\Gamma(k+2,x)}{\Gamma(k+1)} \frac{\Gamma(k+2,y)}{\Gamma(k+1)} \quad (2.20)$$

This awkward sum of sixteen sums over k has to be dealt with, if we are to obtain a definite result.

1.4. *Some new functions are presented*

Let us introduce some new functions:

$$H(x,y)=\sum_{k=0}^{\infty}\psi(k,\rho)\frac{\Gamma(k+1,x)}{\Gamma(k+1)}\frac{\Gamma(k+1,y)}{\Gamma(k+1)} \quad (2.21)$$

$$M(x,y)=\sum_{k=0}^{\infty}\psi(k,\rho)\frac{\Gamma(k+2,x)}{\Gamma(k+1)}\frac{\Gamma(k+1,y)}{\Gamma(k+1)} \quad (2.22)$$

$$J(x,y)=\sum_{k=0}^{\infty}\psi(k,\rho)\frac{\Gamma(k+2,x)}{\Gamma(k+1)}\frac{\Gamma(k+2,y)}{\Gamma(k+1)} \quad (2.23)$$

s.t. $x>0, y>0, 0\le\rho<1$

We first write the coefficients and the variables for each of the respective 16 terms within A_i in columns:

I	II	III	IV	V	VI	VII	VIII
N° of term	**Coef. ($\alpha_i,\beta_i,\gamma_i,\delta_i$)**	**Coef. (u_i,v_i,w_i)**	**First Function (G^1, G^2)**	**First variable x**	**Second function type (G^1, G^2)**	**Second variable y**	**New function (H,M,J)**
1	α_i	$\frac{1-\rho}{\lambda_2}$	G^1	$\frac{qh_i\lambda_1}{1-\rho}$	G^2	$\frac{ph_i\lambda_2}{1-\rho}$	M
2	$-\alpha_i$	$\frac{1-\rho}{\lambda_2}$	G^1	$\frac{ql_i\lambda_1}{1-\rho}$	G^2	$\frac{ph_i\lambda_2}{1-\rho}$	M
3	α_i	$\frac{1-\rho}{\lambda_2}$	G^1	$\frac{qh_i\lambda_1}{1-\rho}$	G^2	$\frac{pl_i\lambda_2}{1-\rho}$	M
4	$-\alpha_i$	$\frac{1-\rho}{\lambda_2}$	G^1	$\frac{ql_i\lambda_1}{1-\rho}$	G^2	$\frac{pl_i\lambda_2}{1-\rho}$	M
5	β_i	$\frac{1-\rho}{\lambda_1}$	G^2	$\frac{qh_i\lambda_1}{1-\rho}$	G^1	$\frac{ph_i\lambda_2}{1-\rho}$	M
6	$-\beta_i$	$\frac{1-\rho}{\lambda_1}$	G^2	$\frac{ql_i\lambda_1}{1-\rho}$	G^1	$\frac{ph_i\lambda_2}{1-\rho}$	M
7	β_i	$\frac{1-\rho}{\lambda_1}$	G^2	$\frac{qh_i\lambda_1}{1-\rho}$	G^1	$\frac{pl_i\lambda_2}{1-\rho}$	M
8	$-\beta_i$	$\frac{1-\rho}{\lambda_1}$	G^2	$\frac{ql_i\lambda_1}{1-\rho}$	G^1	$\frac{pl_i\lambda_2}{1-\rho}$	M
9	γ_i	$\frac{(1-\rho)^2}{\lambda_1\lambda_2}$	G^2	$\frac{qh_i\lambda_1}{1-\rho}$	G^2	$\frac{ph_i\lambda_2}{1-\rho}$	J
10	$-\gamma_i$	$\frac{(1-\rho)^2}{\lambda_1\lambda_2}$	G^2	$\frac{ql_i\lambda_1}{1-\rho}$	G^2	$\frac{ph_i\lambda_2}{1-\rho}$	J
11	γ_i	$\frac{(1-\rho)^2}{\lambda_1\lambda_2}$	G^2	$\frac{qh_i\lambda_1}{1-\rho}$	G^2	$\frac{pl_i\lambda_2}{1-\rho}$	J
12	$-\gamma_i$	$\frac{(1-\rho)^2}{\lambda_1\lambda_2}$	G^2	$\frac{ql_i\lambda_1}{1-\rho}$	G^2	$\frac{pl_i\lambda_2}{1-\rho}$	J
13	δ_i	1	G^1	$\frac{qh_i\lambda_1}{1-\rho}$	G^1	$\frac{ph_i\lambda_2}{1-\rho}$	H
14	$-\delta_i$	1	G^1	$\frac{ql_i\lambda_1}{1-\rho}$	G^1	$\frac{ph_i\lambda_2}{1-\rho}$	H
15	δ_i	1	G^1	$\frac{qh_i\lambda_1}{1-\rho}$	G^1	$\frac{pl_i\lambda_2}{1-\rho}$	H
16	$-\delta_i$	1	G^1	$\frac{ql_i\lambda_1}{1-\rho}$	G^1	$\frac{pl_i\lambda_2}{1-\rho}$	H

Table 1.1: coefficients, functions and variables of each of the 16 terms in Ai

Where the ph_i, pl_i are respectively the upper and lower limits of the segments P_i, and the qh_i and ql_i are respectively the upper and lower limits of the segments Q_i .

To give an idea of how our A_i is now written, by using the appropriate term in each column and summing along the rows we can represent a general term as being of the form:

Coef col.(II) x Coef. Col. (III) x letter_function col(VIII)[x col. (V or VII),y col.(VII or V)]

The letter of the function (M, J, H) is associated with the corresponding variables x or y. However, in the case of the function M, the two variables are not symmetric, so attention must be paid to the place of each: in the first 4 cases (all α's), the x variable and y variable are inverted.

Expressed in α we have

$$\alpha_i \frac{1-\rho}{\lambda_2} M\left(\frac{ph_i\lambda_2}{1-\rho}, \frac{qh_i}{1-\rho}\right) - \alpha_i \frac{1-\rho}{\lambda_2} M\left(\frac{ph_i\lambda_2}{1-\rho}, \frac{ql_i\lambda_1}{1-\rho}\right) + \alpha_i \frac{1-\rho}{\lambda_2} M\left(\frac{pl_i\lambda_2}{1-\rho}, \frac{qh_i\lambda_1}{1-\rho}\right) - \alpha_i \frac{1-\rho}{\lambda_2} M\left(\frac{pl_i\lambda_2}{1-\rho}, \frac{ql_i\lambda_1}{1-\rho}\right)$$

1.5. *Solving the series over k*

To write the functions *H, M, J* in another way that does not include the series over k, we had to call upon the profound resources of Nico Temme[2] who very kindly provided us with the following demonstration:

Let

$$H(x,y,\rho) = (1-\rho)\sum_{k=0}^{\infty} \rho^k Q(k+1,x)Q(k+1,y) \quad (2.24),$$

where $x > 0$, $y > 0$, $-1 < \rho < 1$.

Theorem 1

He demonstrates that:

$$H(x,y,\rho) = e^{-(1-\rho)x} K(\rho x, y) - e^{-(1-\rho)y} K(x, \rho y) + e^{-(1-\rho)y} \quad (2.25),$$

where $K(x,y)$ is the function defined in (2.5) and (2.9) of Temme (1986):

$$K(x,y) = \int_0^x e^{-(t+y)} I_0\left(2\sqrt{ty}\right) dt \quad (2.26).$$

With I_0 the modified Bessel function of the first kind of order zero.

Proof:

Because $Q(m+1,0) = 1$, we have:

$$H(0,0,\rho) = (1-\rho)\sum_{k=0}^{\infty} \rho^k = 1 \quad (2.27).$$

Next, because

$$\frac{d}{dx} Q(k+1,x) = -e^{-x} \frac{x^k}{k!} \quad (2.28),$$

we have

$$\frac{\partial^2 H(x,y,\rho)}{\partial x \partial y} = (1-\rho) e^{-x-y} \sum_{k=0}^{\infty} \rho^k \frac{x^k}{k!} \frac{y^k}{k!} = (1-\rho) e^{-x-y} I_0\left(2\sqrt{\rho xy}\right) \quad (2.29)$$

Also,

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$$
\begin{aligned}
H(0,y,\rho) &= (1-\rho)\sum_{k=0}^{\infty}\rho^k Q(k+1,y) \\
&= (1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k}{k!}\int_y^{\infty} t^k e^{-t}dt \quad (2.30) \\
&= (1-\rho)\int_y^{\infty} e^{-(1-\rho)t}dt \\
&= e^{y(\rho-1)}
\end{aligned}
$$

and because of symmetry:

$$H(x,0,\rho) = e^{x(\rho-1)} \ (2.31).$$

Next,

$$
\begin{aligned}
\frac{\partial H(x,y,\rho)}{\partial x} &= -(1-\rho)e^{-x}\sum_{k=0}^{\infty}\rho^k \frac{x^k}{k!}Q(k+1,y) \\
&= -(1-\rho)e^{-x}\int_y^{\infty} e^{-u} I_0\left(2\sqrt{\rho x u}\right)du \qquad (2.32) \\
&= -(1-\rho)e^{-x}\left[e^{\rho x} - \int_0^y e^{-u} I_0\left(2\sqrt{\rho x u}\right)du\right]
\end{aligned}
$$

Hence, by using (2.30),

$$
\begin{aligned}
H(x,y,\rho) &= -(1-\rho)\int_0^x e^{-(1-\rho)v}dv + e^{-(1-\rho)y} + (1-\rho)\int_0^x\int_0^y e^{-u-v} I_0\left(2\sqrt{\rho uv}\right)dudv \qquad (2.33)\ (35) \\
&= -1 + e^{-(1-\rho)x} + e^{(1-\rho)y} L(x,y,\rho)
\end{aligned}
$$

where $L(x,y,\rho)$ denotes the double integral, which is also given in formula (2.13) of Temme (1986). By using (2.14) of Temme (1986), that is,

$$L(x,y,\rho) = 1 - e^{-(1-\rho)y} + e^{-(1-\rho)y}K(\rho x, y) - e^{-(1-\rho)x}K(y,\rho x) \qquad (2.34)$$

where $K(x,y)$ is given in (28) and the relation

$$K(x,y) + K(y,x) = 1 - e^{-x-y} I_0\left(2\sqrt{xy}\right) \qquad (2.35),$$

we find (2.25). The relation in (2.35) follows from applying the relation in (2.4) of Temme (1986), used also for his own definition of a function $M(y,x)$, which equals $M(x,y)$.

This ends the proof. □

We now deal with the function $M(x,y,\rho)$.

$$M(x,y,\rho) = \sum_{k=0}^{\infty}\psi(k,\rho)\frac{\Gamma(k+2,x)}{\Gamma(k+1)}\frac{\Gamma(k+1,y)}{\Gamma(k+1)} \qquad (2.36)$$

Theorem 2

$$M(x,y,\rho) = \frac{1}{1-\rho}H(x,y,\rho) + \rho\frac{\partial H(x,y,\rho)}{\partial x} - x\frac{\partial H(x,y,\rho)}{\partial x} \qquad (2.37)$$

Proof:

From (11.7) in Temme (1996), we know that:

$$\Gamma(k+2,x) = (k+1)\Gamma(k+1,x) + x^{k+1}e^{-x} \qquad (2.38).$$

Hence

$$M(x,y,\rho) = M_1(x,y,\rho) + M_2(x,y,\rho) \qquad (2.39)$$

where,

$$\begin{aligned} M_1(x,y,\rho) &= (1-\rho)\sum_{k=0}^{\infty}(k+1)\rho^k Q(k+1,x)Q(k+1,y) \\ &= \frac{1}{1-\rho}H(x,y,\rho) + \rho\frac{\partial H(x,y,\rho)}{\partial \rho} \end{aligned} \qquad (2.40)$$

and by using the first form of equation (2.32) and (2.28), we have

$$M_2(x,y,\rho) = -x\frac{\partial H(x,y,\rho)}{\partial x} \qquad (2.41).$$

which completes the proof.□

We finally deal with function $J(x,y,\rho)$

$$J(x,y,\rho) = \sum_{k=0}^{\infty}\psi(k,\rho)\frac{\Gamma(k+2,x)}{\Gamma(k+1)}\frac{\Gamma(k+2,y)}{\Gamma(k+1)} \qquad (2.42)$$

By using (2.38) we obtain

$$J(x,y,\rho) = J_1(x,y,\rho) + J_2(x,y,\rho) + J_3(x,y,\rho) + J_4(x,y,\rho) \quad (2.43)$$

where

$$J_1(x,y,\rho) = (1-\rho)\sum_{k=0}^{\infty}(k+1)^2\rho^k Q(k+1,x)Q(k+1,y), \qquad (2.44)$$

$$J_2(x,y,\rho) = ye^{-y}(1-\rho)\sum_{k=0}^{\infty}(m+1)\frac{\rho^k y^k}{k!}Q(k+1,x), \qquad (2.45)$$

$$J_3(x,y,\rho) = xe^{-x}(1-\rho)\sum_{k=0}^{\infty}(k+1)\frac{\rho^k x^k}{k!}Q(k+1,y), \qquad (2.46)$$

and $J_4(x,y,\rho) = xye^{-x-y}(1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k x^k y^k}{k!k!}$. (2.47)

The function $J_1(x,y)$ follows from differentiating $H(x,y)$ with respect to ρ. This follows from

$$J_1(x,y,\rho) = (1-\rho)\frac{\partial}{\partial\rho}\left[\rho\frac{\partial}{\partial\rho}\left\{\rho\frac{H(x,y,\rho)}{1-\rho}\right\}\right]. \quad (2.48)$$

Next, $J_2(x,y,\rho) = J_3(y,x,\rho)$ and follow from writing

$$\begin{aligned} J_3(x,y,\rho) &= xe^{-x}(1-\rho)\frac{\partial}{\partial\rho}\left[\rho\sum_{k=0}^{\infty}\frac{\rho^k x^k}{k!}Q(k+1,y)\right] \\ &= -x(1-\rho)\frac{\partial}{\partial\rho}\left[\frac{\partial H(x,y,\rho)}{\partial x}\frac{\rho}{1-\rho}\right] \end{aligned}, \quad (2.49)$$

and by differentiating $\partial H(x,y,\rho)/\partial x$. See the first line of (2.32).

$$J_2(x,y,\rho) = ye^{-y}(1-\rho)\frac{\partial}{\partial\rho}\left[\rho\frac{\partial H(x,y,\rho)}{\partial y}\frac{e^y}{1-\rho}\right] \quad (2.50)$$

Finally,

$$J_4(x,y,\rho) = xye^{-x-y}(1-\rho)I_0\left(2\sqrt{\rho xy}\right). \quad (2.51)$$

At last, the A_i terms have been written in terms of the K function defined by N. Temme and the modified Bessel functions without any reference to a series.

We can proceed to write the whole expected cost function as the sum of the A_i terms.

1.6. *Special limit values we need*

For some particular upper and lower limits of the Q_i and P_i segments, such as 0 and infinity, we must calculate the values of our functions.

From (2.35) we have:

$$K(x,x) = \frac{1-e^{-2x}I_0(2x)}{2} \quad (2.52)$$

As $\lim_{x\to\infty} e^{-x}I_0(x) = 1$, we get from (2.52) that $\lim_{x\to\infty} K(x,x) = \frac{1}{2}$ and from (2.26) $\lim_{x\to\infty} K(x,y) = 1$ as well as $\lim_{y\to\infty} K(x,y) = 0$.

The H function is written in terms of sums of K functions and $e^{-x}I_0(x)$. Temme (1996) shows that $\lim_{x\to\infty} e^{-x}I_0(x) = 0$, so that we have

$$\lim_{x\to\infty} H(x,y,\rho) = \lim_{y\to\infty} H(x,y,\rho) = 0.$$

In the same way, *M(x,y,ρ)* can be written as

$$M(x,y,\rho) = \frac{1}{1-\rho}H(x,y,\rho) + (x+\rho y)e^{-x-y}I_0\left(2\sqrt{\rho xy}\right) + \sqrt{\rho xy}\,e^{-x-y}I_1\left(2\sqrt{\rho xy}\right) + xe^{-(1-\rho)x}K(\rho x,y) + \rho ye^{-(1-\rho)y}K(\rho y,x), \quad (2.53)$$

where $I_1\left(2\sqrt{\rho xy}\right)$ is the modified Bessel function of the first kind of order one. We know from Temme(1986) that $I_0(z) \sim \frac{e^z}{\sqrt{2\pi z}}, z\to\infty$. Because we have the condition $\rho < 1$, all terms tend to zero when x and y are large.

Hence:

$$\lim_{x\to\infty} M(x,y,\rho) = \lim_{y\to\infty} M(x,y,\rho) = 0$$

In the same way, when the J function is expanded, we have modified Bessel functions of the first kind of order zero with exponential factors of x and y.

Hence, we conclude that

$$\lim_{x\to\infty} J(x,y,\rho) = \lim_{y\to\infty} J(x,y,\rho) = 0.$$

We also must find the values of these functions when x or y or both are equal to 0.

From (2.30) and the definition of J_1 in (2.48), we have:

$$J_1(0,y,\rho)=\frac{e^{y(\rho-1)}}{(\rho-1)^2}\left[1+\rho\left(1+y(\rho-1)\left\{-3+\rho\left(1+y(\rho-1)\right)\right\}\right)\right].(2.54)$$

In the same way, because of the symmetry of H:

$$J_1(x,0,\rho)=\frac{e^{x(\rho-1)}}{(\rho-1)^2}\left[1+\rho\left(1+x(\rho-1)\left\{-3+\rho\left(1+x(\rho-1)\right)\right\}\right)\right].(2.55)$$

In turn, using (2.30) and (2.31):

$$J_3(0,y,\rho)=J_2(x,0,\rho)=0 \quad (2.56)$$

and

$$J_3(x,0,\rho)=xe^{x(\rho-1)}(1-\rho)(1+x\rho) \quad (2.57)$$

$$J_2(0,y,\rho)=ye^{y(\rho-1)}(1-\rho)(1+y\rho) \quad (2.58)$$

$$J_4(0,y,\rho)=J_4(x,0,\rho)=0 \quad (2.59)$$

Finally for the case when both x and y are equal to 0:

We already have established that $H(0,0,\rho)=1$ from (2.27). We also write:

$J_1(0,0,\rho)=\frac{1+\rho}{(\rho-1)^2}$ (2.60) from the definition of $H(0,y,\rho)$ in (2.30).

$$J_2(0,0,\rho)=J_3(0,0,\rho)=0 \quad (2.61)$$

Function M's behaviour in the vicinity of 0 is straightforward:

$$M(0,y,\rho)=e^{y(\rho-1)}(1-\rho)(1+y\rho) \quad (2.62)$$

$$M(x,0,\rho)=e^{-x(\rho+1)-1}(\rho-1)\left[(x+1)e^{2x\rho+1}+\rho e^{x(\rho+1)+\rho}+\rho(x-1)e^{x+\rho}\right] \quad (2.63)$$

$$M(0,0,\rho)=1-\rho \quad (2.64)$$

1.7. *Expressing A_i*

In each region, we have different values for αi, βi, γi and δi. Let us now recapitulate these, together with the limits of each region for each variable q and p:

Ωi	α_i	β_i	γ_i	δ_i	Lower limit		Upper Limit	
					q	p	q	p
$\Omega 1$	0	$c-\theta_s$	0	$q_L\theta_s$	0	0	qL	∞
$\Omega 2$	$-q_L$	θ_s	1	$q_L(c-\theta_s)$	q_L	0	q_L+q_{La}	$p_{La}-\theta_s$
$\Omega 3$	0	p_{La}	0	$q_L(c-p_{La})$	q_L	$p_{La}-\theta_s$	q_L+q_{La}	$p_{La}+\theta_c$
$\Omega 4$	$-q_L$	$-\theta_c$	1	$q_L(c-\theta_c)$	q_L	$p_{La}+\theta_c$	q_L+q_{La}	∞
$\Omega 5\cup\Omega 8$	$-q_L$	0	1	$q_Lc+q_{La}\theta_s$	q_L+q_{La}	0	∞	$p_{La}-\theta_s$
$\Omega 6\cup\Omega 9$	$-(q_L+q_{La})$	0	1	$q_Lc+q_{La}p_{La}$	q_L+q_{La}	$p_{La}-\theta_s$	∞	$p_{La}+\theta_c$
$\Omega 7\cup\Omega 10$	$-q_L$	0	1	$q_Lc+q_{La}\theta_c$	q_L+q_{La}	$p_{La}+\theta_c$	∞	∞

Table 1.2: coefficients and limits of price and quantity segments in each sub-region

Let us just write the term A_1 over region $\Omega 1$ once the calculations have been done thanks to Mathematica 5.0:

$$\text{Out[110]=}\ \frac{e^{-ql\lambda 2}\left(\theta s\left(-1+ql\lambda 1+e^{ql\lambda 2}(-1+ql\lambda 1)-ql\lambda 2\right)+c\left(1+e^{ql\lambda 2}+ql\lambda 2\right)\right)}{\lambda 1}$$

Annex II

Solving for the optimal parameters of EC_{2-1}.

We must minimize A4+A7+A10. Since these areas in probability space are contiguous, we solve for just one double integral over one region of probability space. The table 1.2 in Annex I of upper and lower limits over *p* and *q* plus the coefficients of *p q* and *pq* (from (22) in 3.6.4) becomes:

Ωi	α_i	β_i	γ_i	δ_i	**Lower limit**		**Upper Limit**	
					q	p	q	p
$\Omega 4 \cup \Omega 7 \cup \Omega 1($	q_L	p_{La}	1	$q_L p_{La}$	q_L	$p_{La}+\theta_c$	∞	∞

Since all upper limits are infinite, this simplifies the calculations as the values of the *H*, *M* and *J* functions tend to zero. And we now can write the total A term using all coefficients as per Table 1.1 of Annex I.

This expression becomes, according to Mathematica 5.0 (where BesselI is the modified Bessel function):

Out[62]=

$$
\begin{aligned}
&\frac{1}{\lambda 2}\Bigg(e^{-(\text{pla}+\theta\text{c})\,\lambda 2+\frac{\text{ql}\,\lambda 1}{-1+\rho}}\,\text{ql}\,\Bigg(e^{\text{pla}\,\lambda 2+\theta\text{c}\,\lambda 2-\frac{\text{ql}\,\lambda 1\,\rho}{-1+\rho}}+e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}\,\text{pla}\,\lambda 2\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]+\\
&e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}\,\theta\text{c}\,\lambda 2\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]+\\
&e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}\,\text{ql}\,\lambda 1\,\rho\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]+\\
&e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\,\text{BesselI}\Big[1,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]-\\
&e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}\,\rho\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\,\text{BesselI}\Big[1,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]+\\
&(1+\text{pla}\,\lambda 2+\theta\text{c}\,\lambda 2)\int_0^{-\frac{(\text{pla}+\theta\text{c})\,\lambda 2\,\rho}{-1+\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{\text{ql}\,t\,\lambda 1}{-1+\rho}}\Big]\,dt+\\
&e^{\frac{(-\text{ql}\,\lambda 1+(\text{pla}+\theta\text{c})\,\lambda 2)\,\rho}{-1+\rho}}\,\text{ql}\,\lambda 1\,\rho\int_0^{-\frac{\text{ql}\,\lambda 1\,\rho}{-1+\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{t\,(\text{pla}+\theta\text{c})\,\lambda 2}{-1+\rho}}\Big]\,dt-\\
&e^{(\text{pla}+\theta\text{c})\,\lambda 2}\int_0^{-\frac{(\text{pla}+\theta\text{c})\,\lambda 2}{-1+\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{\text{ql}\,t\,\lambda 1\,\rho}{-1+\rho}}\Big]\,dt\Bigg)\Bigg)+\\
&e^{-\text{pla}\,\lambda 1-\theta\text{c}\,\lambda 1-\text{ql}\,\lambda 2}\,\text{pla}\,\text{ql}\,\Bigg(e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\int_0^{\frac{\text{ql}\,\lambda 2\,\rho}{1-\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{t\,(\text{pla}+\theta\text{c})\,\lambda 1}{-1+\rho}}\Big]\,dt+\\
&e^{\text{ql}\,\lambda 2}\Bigg(1-e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\int_0^{\frac{\text{ql}\,\lambda 2}{1-\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{t\,(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\Big]\,dt\Bigg)\Bigg)+\\
&\frac{1}{\lambda 1\,(-1+\rho)}\Bigg(e^{\frac{\text{ql}\,\lambda 2+\text{pla}\,\lambda 1\,(1+\rho)+\theta\text{c}\,\lambda 1\,(1+\rho)}{-1+\rho}}\,\text{pla}\,(1-\rho)\\
&\Bigg(-e^{-\frac{(\text{pla}\,\lambda 1+\theta\text{c}\,\lambda 1+\text{ql}\,\lambda 2)\,\rho}{-1+\rho}}\,(1+\text{ql}\,\lambda 2)\int_0^{\frac{\text{ql}\,\lambda 2\,\rho}{1-\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{t\,(\text{pla}+\theta\text{c})\,\lambda 1}{-1+\rho}}\Big]\,dt+\\
&e^{-\frac{\text{ql}\,\lambda 2+2\,(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\Bigg(-1-e^{\frac{\text{ql}\,\lambda 2+(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\,\text{ql}\,\lambda 2\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]-\\
&e^{\frac{\text{ql}\,\lambda 2+(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\,\text{pla}\,\lambda 1\,\rho\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]-\\
&e^{\frac{\text{ql}\,\lambda 2+(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\,\theta\text{c}\,\lambda 1\,\rho\,\text{BesselI}\Big[0,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]-\\
&e^{\frac{\text{ql}\,\lambda 2+(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\,\text{BesselI}\Big[1,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]+\\
&e^{\frac{\text{ql}\,\lambda 2+(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\,\rho\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\,\text{BesselI}\Big[1,\,2\sqrt{\frac{\text{ql}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\lambda 2\,\rho}{(-1+\rho)^2}}\Big]-\\
&e^{\frac{\text{ql}\,\lambda 2}{-1+\rho}}\,(\text{pla}+\theta\text{c})\,\lambda 1\,\rho\int_0^{-\frac{(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{\text{ql}\,t\,\lambda 2}{-1+\rho}}\Big]\,dt+e^{\frac{(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\\
&\int_0^{\frac{\text{ql}\,\lambda 2}{1-\rho}}e^{-t}\,\text{BesselI}\Big[0,\,2\sqrt{-\frac{t\,(\text{pla}+\theta\text{c})\,\lambda 1\,\rho}{-1+\rho}}\Big]\,dt\Bigg)\Bigg)\Bigg)+\frac{(-1+\rho)^2\,\text{J}\Big[\frac{\text{ql}\,\lambda 2}{1-\rho},\,-\frac{(\text{pla}+\theta\text{c})\,\lambda 1}{-1+\rho},\,\rho\Big]}{\lambda 1\,\lambda 2}
\end{aligned}
$$

As can be seen the resulting equation has no evident optimal solutions.

Numerical example

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